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RENDERING

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12
Radiometry

One of important aspects of physically-based rendering is to simulate physical interactions between lights and materials in a correct manner. To explain these physical interactions, we discuss various physical models of light in this chapter. Most rendering effects that we observe can be explained by a simple, geometric optics. Based on this simple light model, we then explain radiometric quantities that are important for computing colors. Finally, we explain basic material models that are used for simulating the physical interaction with lights.

12.1 Physics of Light

Understanding light has drawn major human efforts in physics and resulted in many profound progress on optics and related fields. Light or visible light is a type of electromagnetic radiations or waves that we can see through our eyes. The most general physical model is based on quantum physics and explains the duality of wave and particle natures of light.

While the quantum physics explains the mysterious wave-particle duality, it is rather impossible to simulate the quantum physics for making our applications, i.e., games and movies, at the current computing hardware. One of simpler light models is the wave model that treats light like sound. Such wave characteristics become prominent, when the wavelength of light is similar to sizes of interacting materials, and diffraction is one of such phenomena. For example, when we see sides of CD, we can see rainbow-like color patterns, which are created by small features of the CD surface.

The most commonly used light model used in computer graphics so far is the geometric optics, which treats light propagation as rays. This model assumes that object sizes are much bigger than the wavelength of light, and thus wave characteristics disappear mostly. This geometric optics can support reflection, refraction, etc.

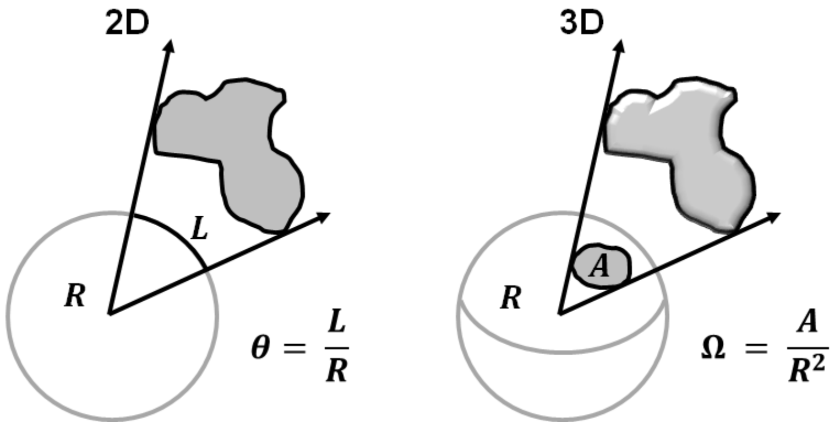

Figure 12.1: Solid angles in 2 D and 3 D cases.

Many rendering methods based on ray tracing assumes the geometric optics, and we also assume this model unless mentioned otherwise.

Our goal is then to measure the amount of energy that a particular ray carries or that a particular location receives from. Along this line, we commonly use a hemisphere, specifically, hemispherical coordinates, to parameterize rays that can arrive at a particular location in a surface. We discuss hemispherical coordinates before we move on to studying radiometry.

Solid angles. We use the concept of solid angles for various integration on the hemisphere. The solid angle is used to measure how much an objected located in 3 D space affects a point in a surface. This metric is very useful for computing shadow and other factors related to visibility. In the 2 D case (the left figure of Fig. 12.1), a solid angle, Ω, of an object is measured by $\frac{L}{R}$, where L is the length of the arc, where the object is projected to in the 2 D hemisphere (or sphere). R is the radius of the sphere; we typically use a unit sphere, where $R = 1$. The unit of the solid angle in the 2 D case is measured by radians. The solid angle mapping to the full circle is 2π radians.

The solid angle in the 3 D case is computed by $\frac{A}{R^2}$, whose unit is steradians (the right figure of Fig. 12.1). A indicates the area subtended by the 3 D object in the hemisphere. For example, the full sphere has 4π steradians.

Hemispherical coordinates. A hemisphere is two dimensional surface and thus we can represent a point on the hemisphere with two parameters such as latitude, θ, and longitude, φ (Fig. 12.2), where $\theta \in [0, \frac{\pi}{2}]$ and $\varphi \in [0, 2\pi]$. Now let's see how we can compute the differential area, dA, on the hemisphere controlled by $d\phi$ and $d\theta$.

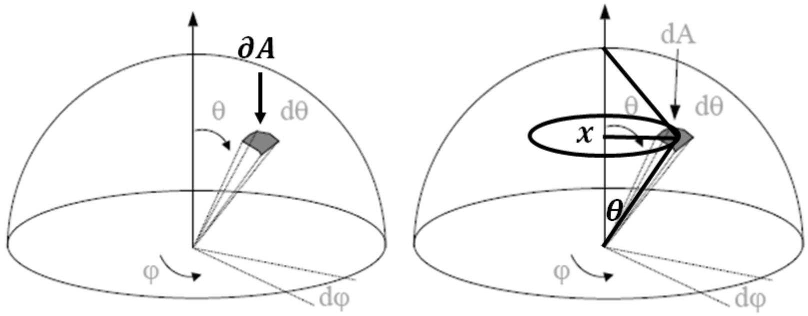

Figure 12.2: Hemispherical coordinates (θ, φ). These images from slides of Kavita Bala.

In infinitely small differential angles, we can treat that the area is approximated by a rectangular shape, whose area can be computed by multiplying its height and width. Its height is given by $d\theta$. On the other hand, its width varies depending on θ; its largest and minimum occur at $\theta = \pi/2$ and $\theta = 0$, respectively.

To compute the width, we consider a virtual circle that touches the rectangular shape of the hemisphere. Let x be the radius of the ch ircle. The radius is then compute by $\sin\theta = \frac{x}{r}, x = r\sin\theta$, where r is the radius of the hemisphere. The width is then computed by applying the concept of the solid angle, and is $r\sin\theta d\phi$. We then have the following differentials:

$$dA = (r\sin\theta d\phi)(rd\theta). \tag{12.1}$$

Based on this equation, we can easily derive differential solid angles, dw:

$$dw = \frac{dA}{r^2} \tag{12.2}$$

$$= \sin\theta d\phi d\theta. \tag{12.3}$$

We use these differential units to define the rendering equation (Ch. 13.1).

12.2 Radiometry

In this section, we study various radiometric quantities that are important for rendering. Human perception on brightness and colors depends on various factors such as the sensitivity of photoreceptor cells in our eyes. Nonetheless, those photoreceptor cells receive photons and trigger biological signals. As a result, measuring photons, i.e., energy, is the first step for performing the rendering process.

Power or flux. Power, P, is a total amount of energy consumed per unit time, denoted by dW/dt, where W indicates watt. In our

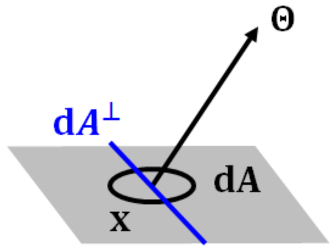

Figure 12.3: Radiance is measured per unit projected area, $dA^\perp$, while we receive the energy on the surface A.

rendering context, it is the total amount of energy arriving at (or passing through) a surface per unit time, and also called radiant flux. Its unit is Watt, which is joules per second. For example, we say that a light source emits 50 watts of radiant power or 20 watts of radiant power is incident on a table.

Irradiance or radiosity. Irradiance is power or radiant flux arriving at a surface per unit area, denoted by dW/dA with the unit of W/m^2. Radiant exitance is the radiant flux emited by a surface per unit area, while radiosity is the radiant flux emitted, reflected, or transmitted from a surface per unit area; that is why the radiosity algorithm has its name (Ch. 11). For example, when we have a light source emitting $100W$ of area $0.1m^2$, we say that the radiant exitance of the light is $1000W/m^2$.

Radiance is one of the most important radiometric quantity used for physically-based rendering.

Radiance. In terms of computing rendering images, computing the radiance for a ray is the most important radiometric measure. The radiance is radiant flux emitted, reflected, or received by a surface per unit solid angle and per unit projected area, $dA^\perp$, whose normal is aligned with the center of the solid angle (Fig. 12.3):

$$L(x \to \Theta) = \frac{d^2P}{d\Theta dA^\perp} \quad (12.4)$$

$$= \frac{d^2P}{d\Theta dA\cos\theta}. \quad (12.5)$$

$\cos\theta$ is introduced for considering the projected area.

Diffuse emitter. Suppose that we have an ideal diffuse emitter that emits the equal radiance, L, in any possible direction. Its irradiance on a location is measured as the following:

$$\begin{aligned} E &= \int_\Theta L\cos\theta dw_\Theta, \\ &= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} L\cos\theta\sin\theta d\theta d\phi = \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} L\cos\theta\sin\theta d\theta \\ &= 2\pi L\frac{1}{2} = L\pi. \end{aligned} \quad (12.6)$$

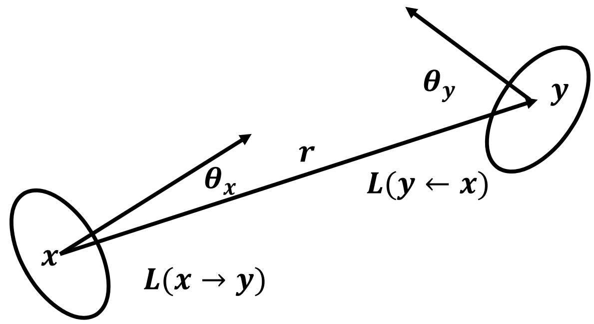

Figure 12.4: A geometric configuration between two patches.

where Θ is the hemispherical coordinates, (θ, ϕ).

Invariance property of radiance. We intuitively know that radiance moving from a location to another location along a straight line does not vary as long as there are neither incoming nor exiting energy along the line. We can prove this property by assuming the conservation of the energy. For example, suppose a geometric configuration shown in Fig. 12.4.

Radiance does not vary along a straight ray, as long as any additional energy does not enter or exit along the ray.

We would like to show that the radiance from a location x to another location y, $L(x \rightarrow y)$, is equal to the radiance that we observe at the location y in the angle from the location x, $L(y \leftarrow x)$. We then have the following theorem:

Theorem 12.2.1. *Under the energy conservation, $L(x \rightarrow y) = L(y \leftarrow x)$, when there is nor incoming and or exiting energy on the line between x and y.*

Proof. By the definition of the radiance, we have the following equations:

$$L(x \rightarrow y) = \frac{d^2P}{dA_x \cos\theta_x dw_{x\rightarrow y}},$$
$$L(y \leftarrow x) = \frac{d^2P}{dA_y \cos\theta_y dw_{y\rightarrow x}}, \quad (12.7)$$

where $dw_{x\rightarrow y}$ is the solid angle of the differential unit area at y, dA_y observed from x, and $dw_{y\rightarrow x}$ is defined similarly, i.e., the solid angle of the differential unit area at x, dA_x, observed from y. Since these two equations have the same energy term, d^2P, we have the following equality:

$$L(x \rightarrow y) dA_x \cos\theta_x dw_{x\rightarrow y} = L(y \leftarrow x) dA_y \cos\theta_y dw_{y\rightarrow x}. \quad (12.8)$$

By utilizing the definition of the solid angle, we then have the follow-

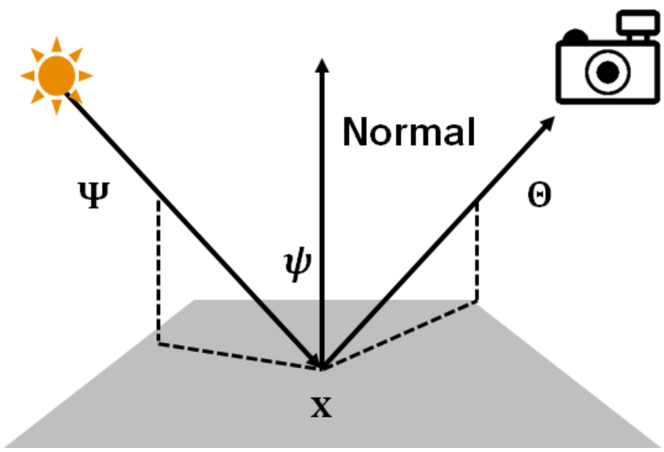

Figure 12.5: A configuration setting for measuring the BRDF is shown. Ψ and Θ are incoming and outgoing directions, while ψ is the angle between the surface normal and Ψ.

ing equality:

$$L(x \rightarrow y) dA_x \cos\theta_x \frac{dA_y \cos\theta_y}{r^2} = L(y \leftarrow x) dA_y \cos\theta_y \frac{dA_x \cos\theta_x}{r^2},$$
$$L(x \rightarrow y) = L(y \leftarrow x). \quad (12.9)$$

□

12.3 Materials

We discussed the Snell's law to support the ideal specular (Sec. 10.1. Phong illumination supports ideal diffuse and a certain class of glossy materials (Ch. 8). However, some materials have complex appearances that are not captured by those ideal specular, ideal diffuse, and glossy materials. In this section, we discuss Bidirectional Reflectance Distribution Function (BRDF) that covers a wide variety of materials.

Our idea is to measure an appearance model of a material and to use it within physically based rendering methods. Suppose the light and camera settings shown in Fig. 12.5. We would like to measure how the material reflects incoming radiance with a direction of Ψ into outgoing radiance with a direction of Θ. As a result, BRDF, $f_r(x, \Psi \rightarrow \Theta)$, at a particular location x is a four dimensional function, defined as the following:

$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos\psi dw_\Psi}, \quad (12.10)$$

where ψ is the angle between the normal of the surface at x and the incoming direction Ψ, and dw_Ψ is the differential of the solid angle for the light. The main reason why we use differential units, not non-differential units, is that we want to cancel existing light energy in addition to the light used for measuring the BRDF.

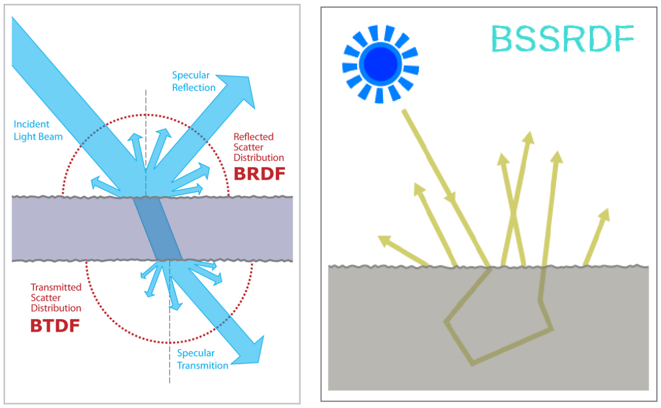

Figure 12.6: These images show interactions between the light and materials that BRDF, BTDF, and BSSRDF. These images are excepted from Wiki.

The BRDF satisfies the following properties:

1. Reciprocity. Simply speaking, when we switch locations of the camera and light, we still get the same BRDF. In other words, $f_r(x, \Psi \to \Theta) = f_r(x, \Theta \to \Psi)$.

2. Energy conservation. $\int_\Theta f_r(x, \Psi \to \Theta) \cos \theta dw_\Theta \leq 1$.

To measure a BRDF of a material, a measuring device, called gonioreflectometer, is used. Unfortunately, measuring the BRDF takes long time, since we have to scan different incoming and outgoing angles. Computing BRDFs in an efficient manner is an active research area.

Material appearance varies depending on wavelengths of lights. To support such material appearance depending on wavelengths of lights, we can measure BRDFs as a function of wavelengths, and use a BRDF given a wavelengths of the light.

12.3.1 *Other Distribution Functions*

So far, we mainly considered BRDF. BRDF, however, cannot support many other rendering effects such as subsurface scattering.

BRDF considered reflection at a particular point, x. For translucent models, lights can pass through the surface and are reflected in the other side of the surface. To capture such transmittance, BTDF (Bi-direction Transmittance Distribution Function) is designed (Fig. 12.6). Furthermore, light can be emitted from points other than the point x that we receive the light. This phenomenon occurs as a result of transmittance and reflection within a surface of translucent materials. BSSRDF (Bidirectional Surface Scattering Reflection Distribution Function) captures such complex phenomenon. Capturing and rendering these complex appearance models is very important topics and still an active research area.