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# CS686: Probabilistic Roadmaps

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(윤성익)

Course URL:  
<http://sgvr.kaist.ac.kr/~sungeui/MPA>

**KAIST**

The KAIST logo consists of the letters "KAIST" in a bold, blue, sans-serif font. Below the text is a light blue, horizontal oval shape that serves as a shadow or base for the letters.

# Announcements

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- **Mid-term exam**
  - **closed book: simple reviews on lecture materials)**
  - **11am on Oct-18 (Wed) at the class time**

# Deadlines

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- **Declare project team members**
  - **By 9/26 at KLMS**
- **Declare two papers for student presentations**
  - **First come, first served**
  - **Paper title, conf. name, publication year**
  - **by 10/10 at KLMS**
  - **Choose papers from 2019 ~ now, published on top-tier conf./journals**
- **Student presentations will start right after the mid-term exam**
  - **2 talks per each class; 15 min for each talk**
  - **Each presenter needs two short quiz**

# Project Guidelines: Project Topics

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- **Any topics related to the course theme are okay**
  - **You can find topics by browsing recent papers**
- **You can bring your own research to the class, only if it is related to the course theme**
  - **You need to get a permission from me for this**

# Expectations

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- **Mid-term project presentation**
  - **Introduce problems and explain why it is important**
  - **Give an overall idea on the related work**
  - **Explain what problems those existing techniques have**
  - **(Optional) explain how you can address those problems, and try out existing codes**
  - **Explain roles of each member**

# Expectations

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- **Final-term project presentation**
  - **Review materials that you talked for your mid-term project**
  - **Present your ideas that can address problems of those state-of-the-art techniques**
  - **Give your qualitatively (or intuitive) reasons how your ideas address them**
  - **Also, explain expected benefits and drawbacks of your approach**
  - **(Optional) backup your claims with quantitative results collected by some implementations**
  - **Explain roles of each member**

# A few more comments

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- **Start to implement a paper, if you don't have any clear ideas**
  - **While you implement it, you may get ideas about improving it**
- **Or, find codes that you can start with**

# Final-project evaluation sheet

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You name:

ID:

Score table: higher score is better.

Speaker	Novelty of the project and idea (1 ~ 5)	Practical benefits of the method (1 ~ 5)	Completeness level of the project (1 ~ 5)	Total score (3 ~ 15)	Role of each student is clear and well balanced? (Yes or No)
XXX					
YYY					



# Class Objectives

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- **Understand probabilistic roadmap (PRM) approaches**
  - **Multi-query PRMs**
  
- **Last time:**
  - **Proximity queries: collision detection and distance computation**

# Difficulty with Classic Approaches

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- **Running time increases exponentially with the dimension of the configuration space**
  - **For a  $d$ -dimension grid with 10 grid points on each dimension, how many grid cells are there?**

$$10^d$$

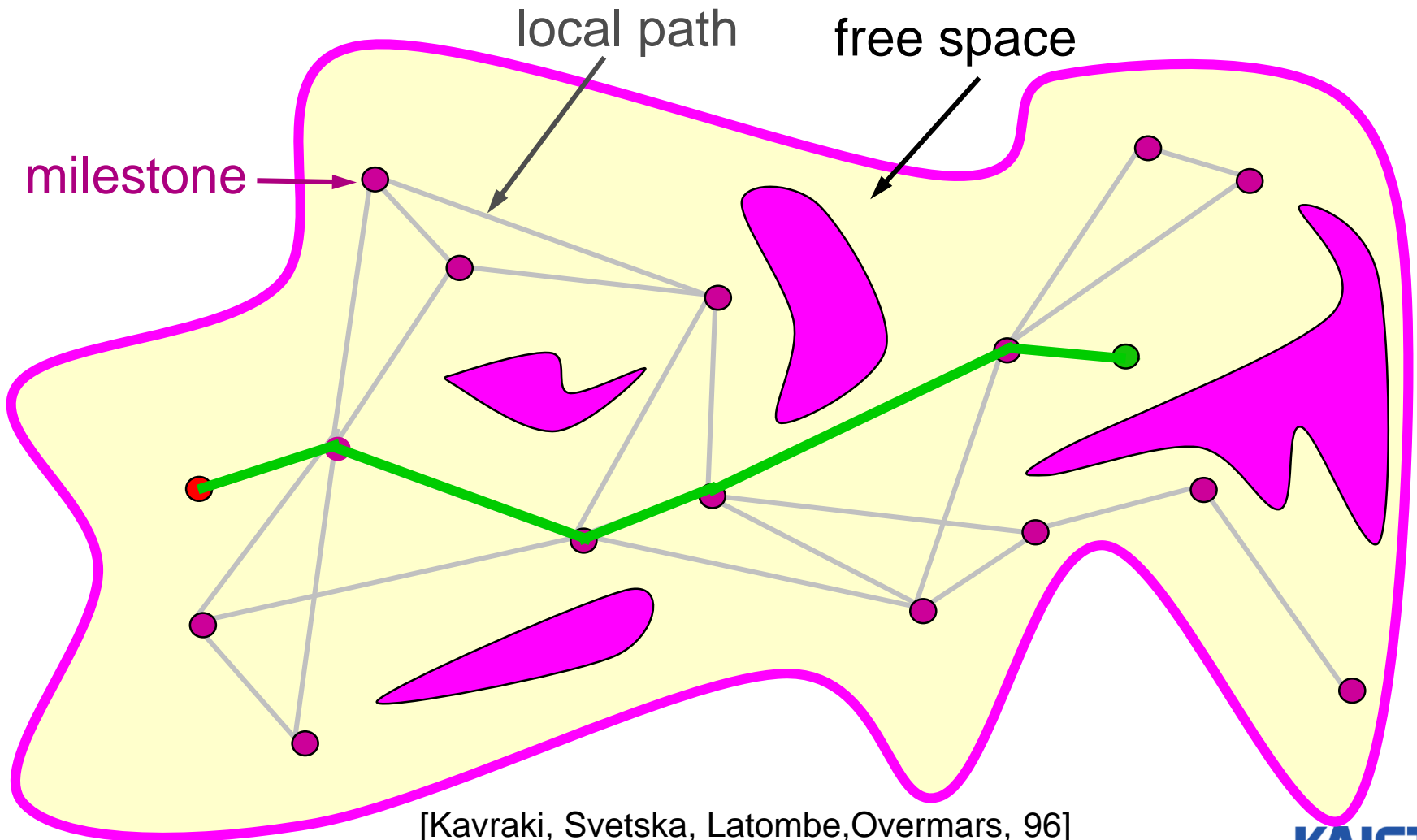
- **Several variants of the path planning problem have been proven to be PSPACE-hard**

# Completeness

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- **Complete algorithm → Slow**
  - A **complete** algorithm finds a path if one exists and reports no otherwise
  - Example: Canny's roadmap method
- **Heuristic algorithm → Unreliable**
  - Example: potential field
- **Probabilistic completeness**
  - Intuition: If there is a solution path, the algorithm will find it with high probability

# Probabilistic Roadmap (PRM): multiple queries

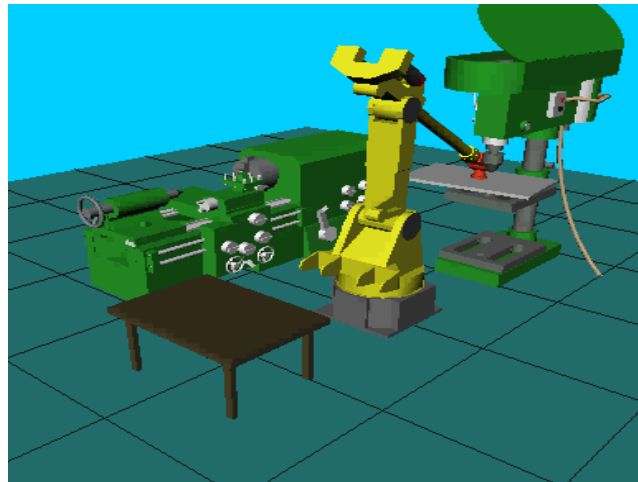


[Kavraki, Svetska, Latombe, Overmars, 96]

# Assumptions

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- **Static obstacles**
- **Many queries to be processed in the same environment**
- **Examples**
  - **Navigation in static virtual environments**
  - **Robot manipulator arm in a workcell**



# Overview

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- **Precomputation: roadmap construction**
  - **Uniform sampling**
  - **Resampling (expansion)**
- **Query processing**

# Uniform sampling

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**Input:** geometry of the moving object & obstacles

**Output:** roadmap  $G = (V, E)$

```
1:  $V \leftarrow \emptyset$  and  $E \leftarrow \emptyset$ .
2: repeat
3:    $q \leftarrow$  a configuration sampled uniformly at random from  $C$ 
4:   if CLEAR( $q$ ) then
5:     Add  $q$  to  $V$ .
6:      $N_q \leftarrow$  a set of nodes in  $V$  that are close to  $q$ .
6:     for each  $q' \in N_q$ , in order of increasing  $d(q, q')$ 
7:       if LINK( $q', q$ ) then
8:         Add an edge between  $q$  and  $q'$  to  $E$ .
```

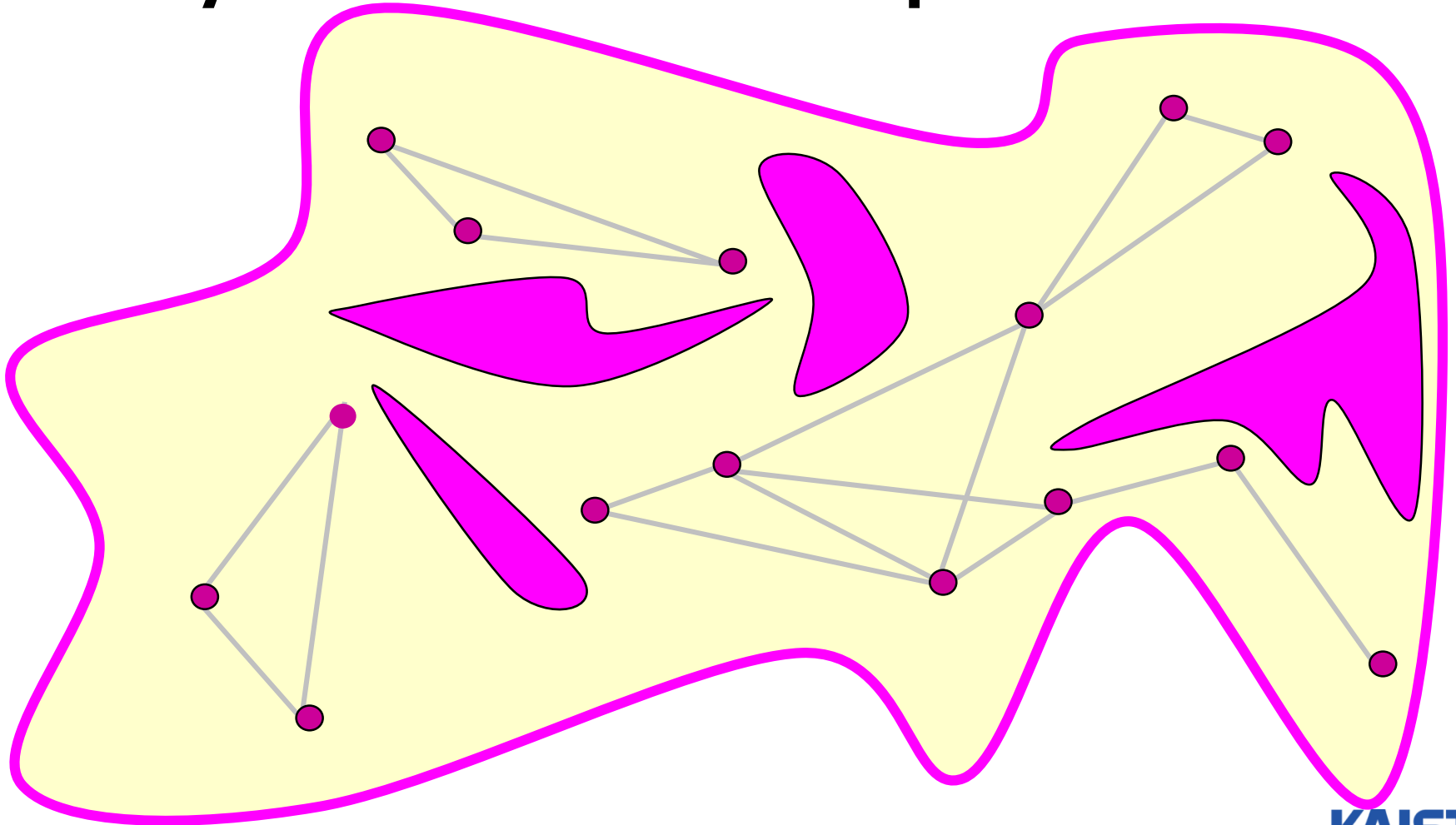
The graph  $G$  is called a **probabilistic roadmap**

The nodes in  $G$  are called **milestones**

# Difficulty

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- Many small connected components





# Resampling (expansion)

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- **Failure rate**

$$r(q) = \frac{\text{\#. failed LINK}}{\text{\#. LINK}}$$

- **Normalized weight**

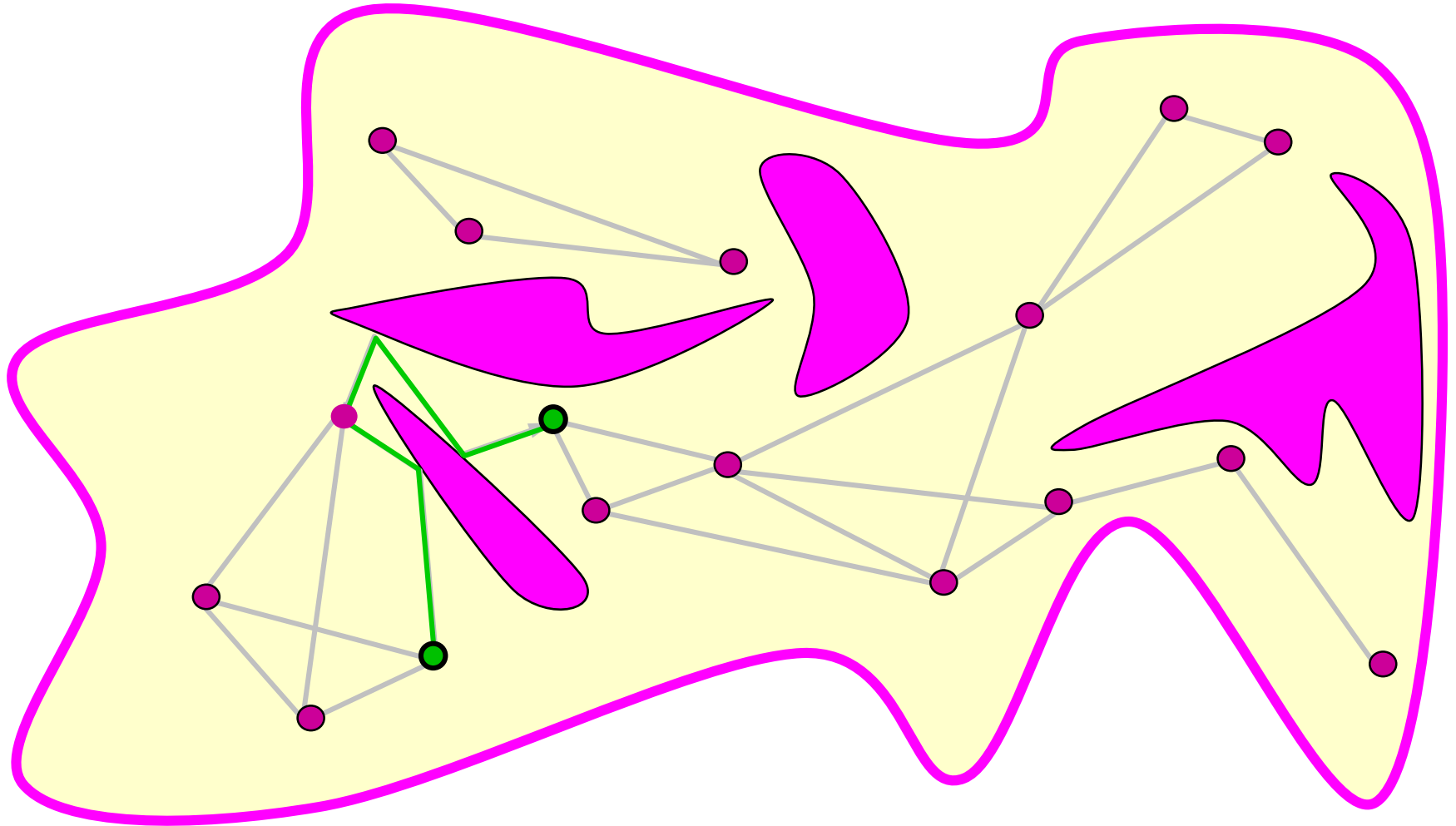
$$w(q) = \frac{r(q)}{\sum_p r(p)}$$

- **Resampling probability**

$$\Pr(q) = w(q)$$

# Resampling (expansion)

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# Query processing

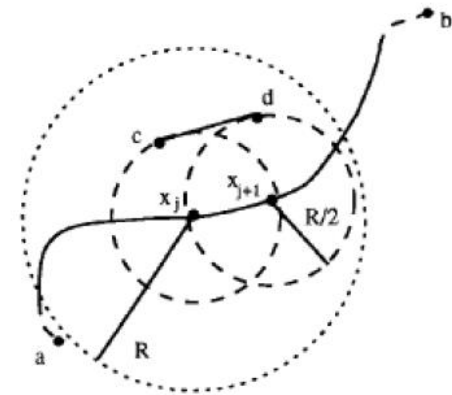
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- **Connect  $q_{init}$  and  $q_{goal}$  to the roadmap**
- **Start at  $q_{init}$  and  $q_{goal}$ , perform a random walk, and try to connect with one of the milestones nearby**
- **Try multiple times**

# Error

- If a path is returned, the answer is always correct
- If no path is found, the answer may or may not be correct. We hope it is correct with high probability.
  - Refer to Theoretical Analysis of my draft

$$P(\text{Fail}) \leq \frac{2L}{R} \exp(-\alpha_D R^D N).$$

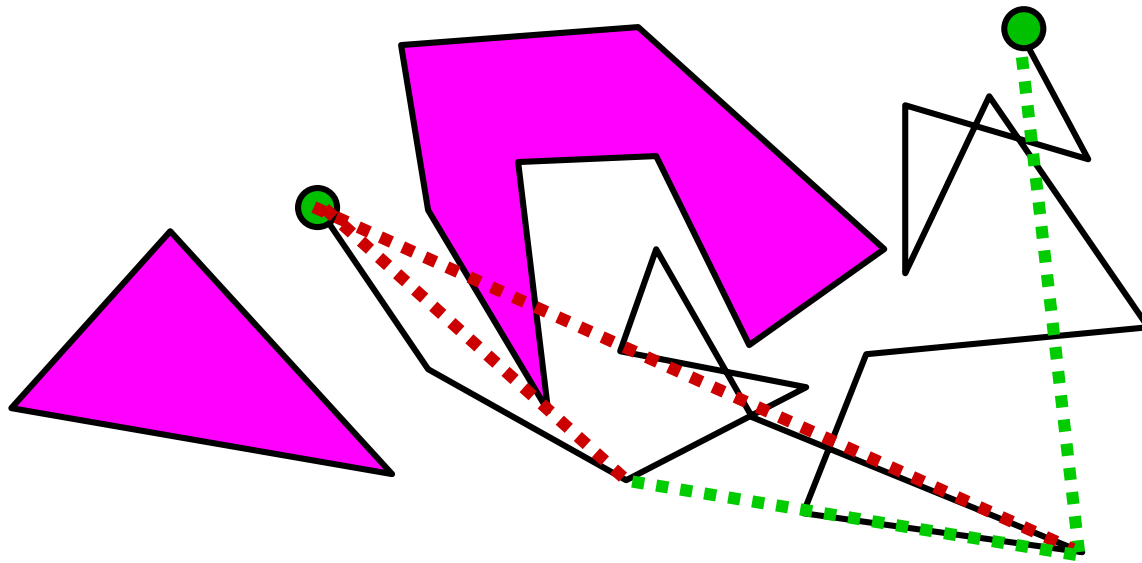


L: path lengths, N: # of samples, D is dimension  
R: the clearance between the robot and obstacles

$$\alpha_D = 2^{-D} \frac{\pi^{D/2}}{\Gamma(D/2+1) \text{Vol}(C_{\text{free}})}$$

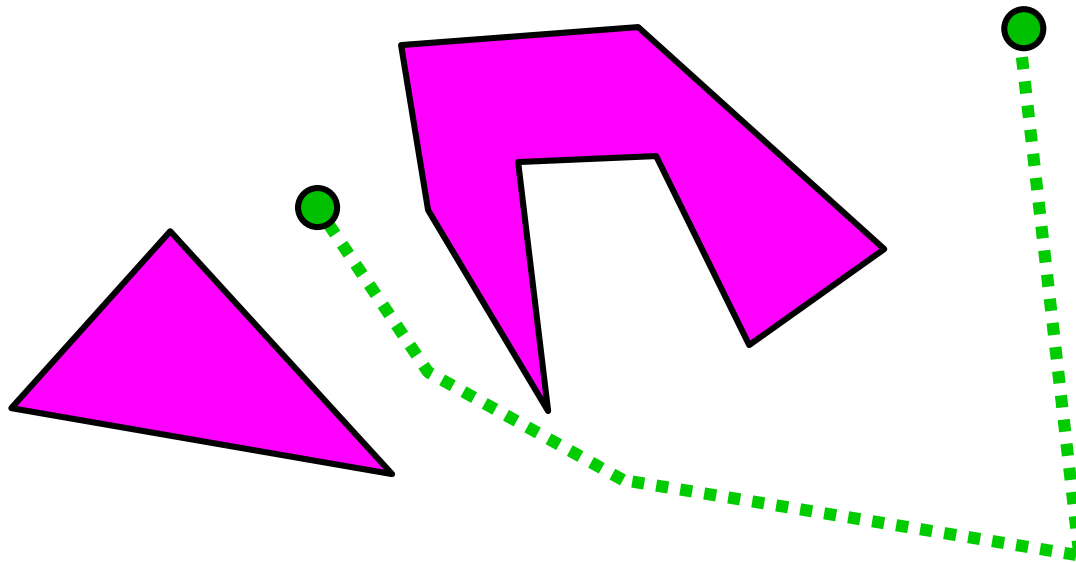
# Smoothing the path

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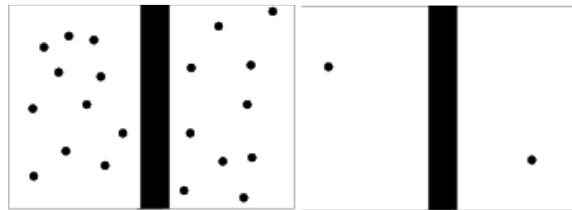
# Smoothing the path

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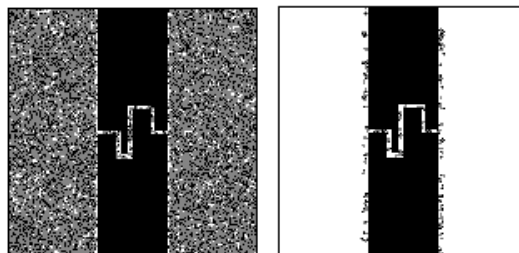


# Sampling Strategies

- **Visibility-based Probabilistic roadmaps for Motion planning**



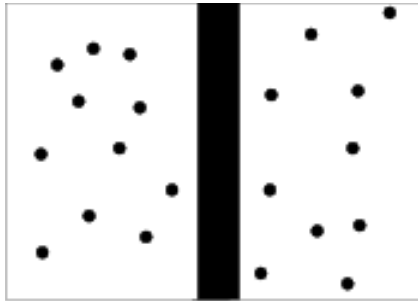
- **The Gaussian Sampling Strategy for PRM's**
  - *Sample near the boundaries of the C-space obstacles with higher probability*



# Visibility-based PRM

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- Computes a very compact roadmap



*Classical PRM*



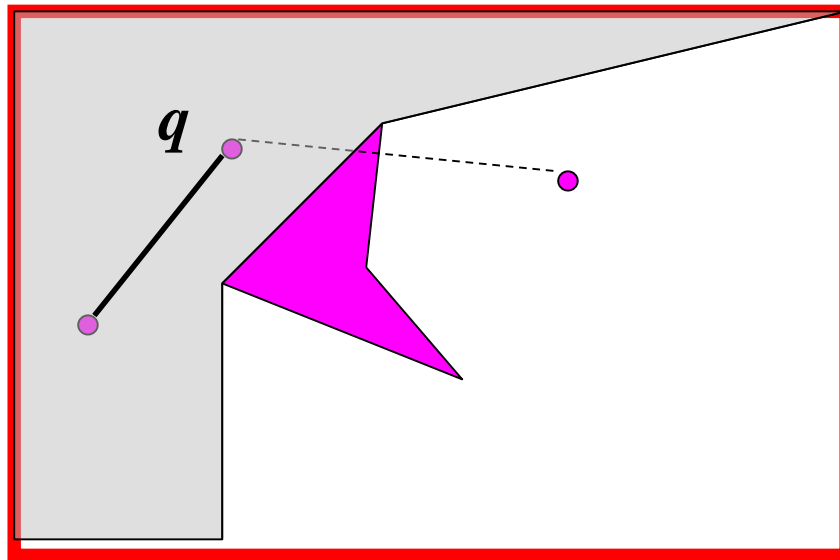
*Visibility roadmap*



# Visibility Domain

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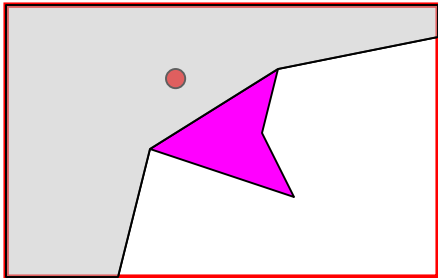
- **Visibility domain of a free configuration  $q$ :**
  - The grey region



# Guard Nodes

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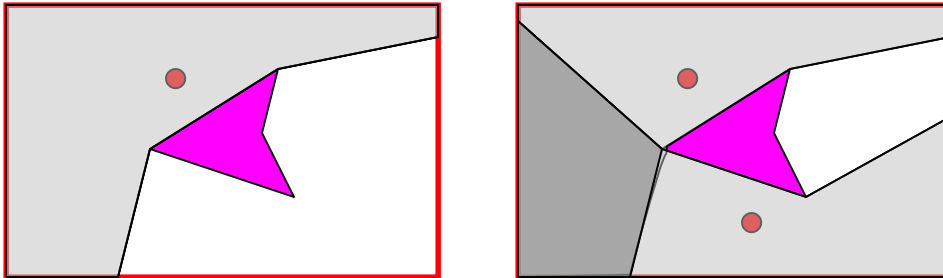
- The C-space fully captured by 'guard' nodes



# Guard Nodes

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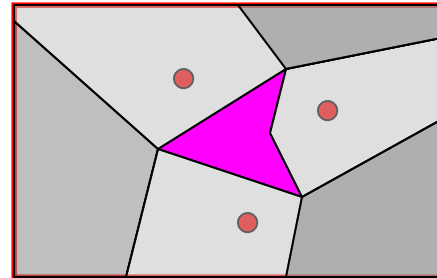
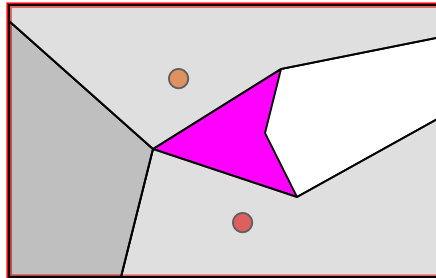
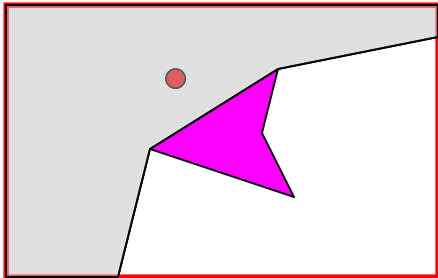
- The C-space fully captured by 'guard' nodes.



# Guard Nodes

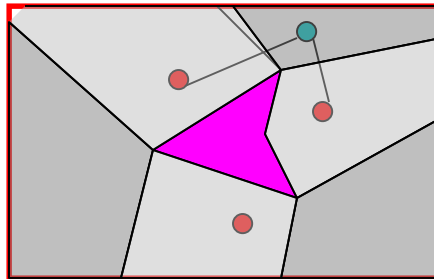
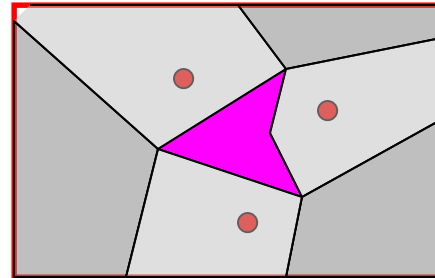
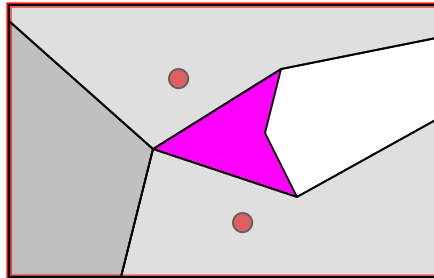
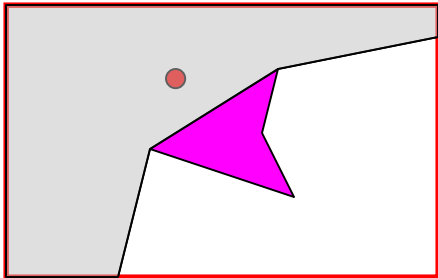
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- The C-space fully captured by 'guard' nodes.



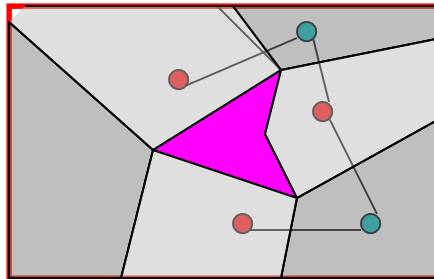
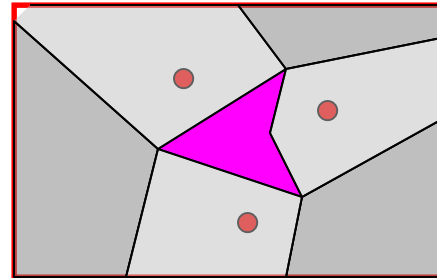
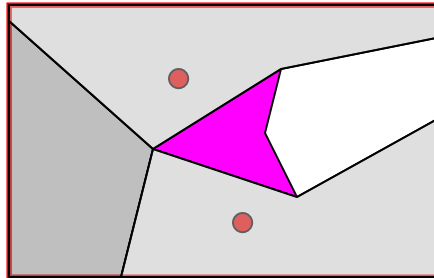
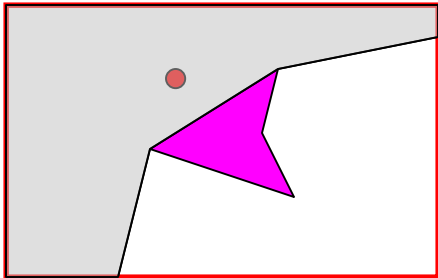
# Connection Nodes

- The C-space being captured by 'guards' and 'connection' nodes.



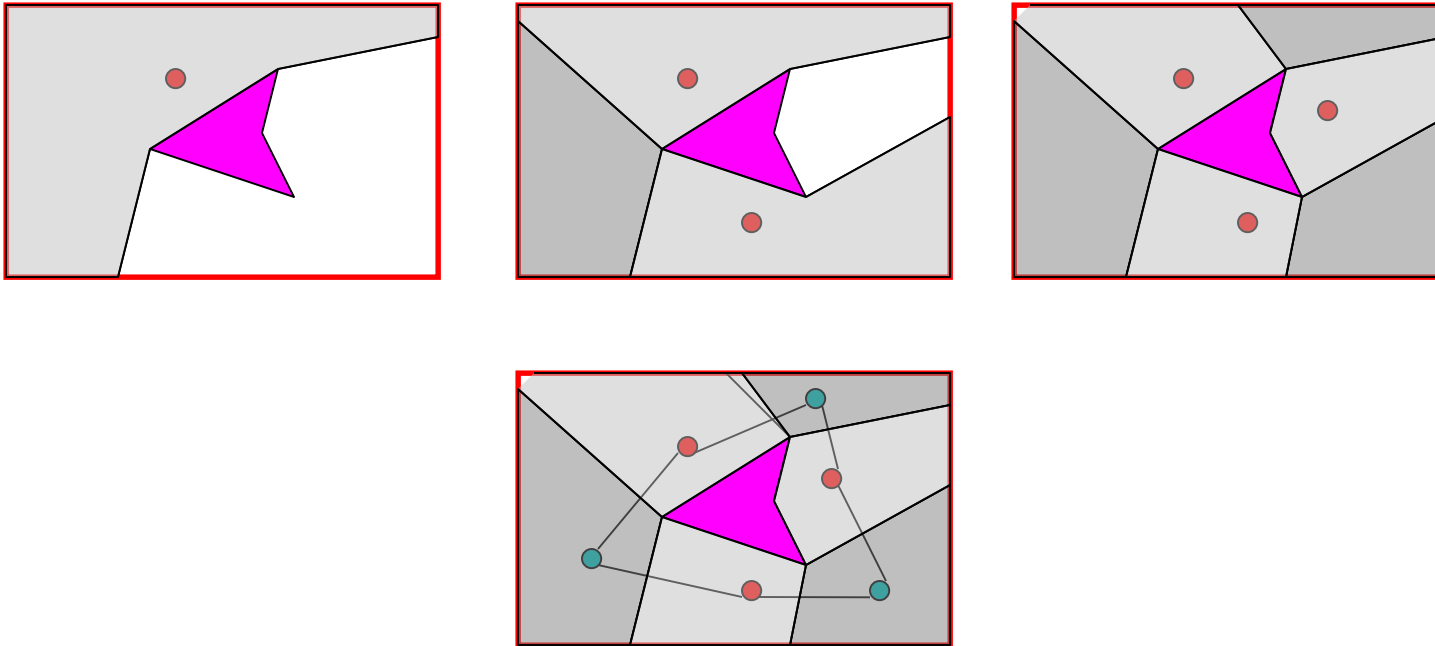
# Connection Nodes

- The C-space being captured by 'guards' and 'connection' nodes.



# Connection Nodes

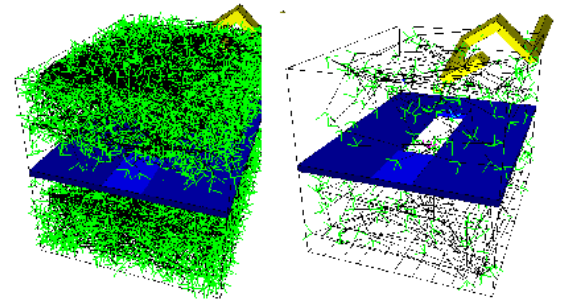
- The C-space fully captured by 'guards' and 'connection' nodes.



- We do not need any other additional node in the roadmap

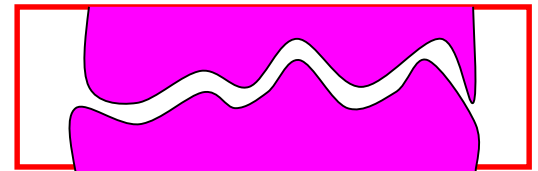
# Remarks

- **Maintains a very compact roadmap, resulting in faster query time**



- **But:**

- **There is a tradeoff with high cost of processing each new milestone**
- **The problem of capturing the narrow passage effectively is still the same as in the basic PRM.**





# Summary

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- **What probability distribution should be used for sampling milestones?**
- **How should milestones be connected?**
- **A path generated by a randomized algorithm is usually jerky. How can a path be smoothed?**
  
- **Single-query PRMs were proposed, but RRT techniques are more widely used**

# Class Objectives were:

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- **Understand probabilistic roadmap (PRM) approaches**
  - **Multi-query PRMs**

# Next Time..

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- **RRT techniques and their recent advancements**

# Homework for Every Class

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- **Submit summaries of 2 ICRA/IROS/RSS/CoRL/TRO/IJRR papers**
- **Go over the next lecture slides**
- **Come up with two question submissions before the mid-term exam**