CS686: Configuration Space I

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Course URL:
http://sglab.kaist.ac.kr/~sungeui/MPA
Announcements

● Make a project team of 2 or 3 persons for your final project
  ● Each student has a clear role
  ● Declare team members at KLMS by Apr-3; you don’t need to define the topic by then

● Each student
  ● Present two papers related to the project
  ● 15 min ~ 20 min for each talk

● Each team
  ● Give a mid-term review presentation for the project
  ● Give the final project presentation
Tentative schedule

See the homepage
Class Objectives

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics
What is a Path?

A box robot

Linked robot
Rough Idea of C-Space

- Convert rigid robots, articulated robots, etc. into points
- Apply algorithms in that space, in addition to the work space
Mapping from the Workspace to the Configuration Space
Configuration Space

- Definitions and examples
- Obstacles
- Paths
- Metrics
The configuration of an object is a complete specification of the position of every point on the object.

- Usually a configuration is expressed as a vector of position & orientation parameters: \( q = (q_1, q_2, \ldots, q_n) \)

The configuration space \( C \) is the set of all possible configurations.

- A configuration is a point in \( C \)

C-space formalism: Lozano-Perez ‘79
Examples of Configuration Spaces
Examples of Configuration Spaces

Consider the end-effector in the workspace?

This is not a valid C-space!
Examples of Configuration Spaces

The topology of $C$ is usually **not** that of a Cartesian space $R^n$.

$S^1 \times S^1 = T^2$
Examples of Circular Robot

(a) (b) (c)
Dimension of Configuration Space

- The dimension of the configuration space is the minimum number of parameters needed to specify the configuration of the object completely.

- It is also called the number of degrees of freedom (dofs) of a moving object.
Example: Rigid Robot in 2-D Workspace

- **3-parameter specification:** $q = (x, y, \theta)$ with $\theta \in [0, 2\pi)$.
  - 3-D configuration space
Example: Rigid Robot in 2-D workspace

- **4-parameter specification:** \( q = (x, y, u, v) \) with \( u^2 + v^2 = 1 \). **Note** \( u = \cos \theta \) and \( v = \sin \theta \)

- **dim of configuration space = 3**
  - Does the dimension of the configuration space (number of dofs) depend on the parametrization?
Holonomic and Non-Holonomic Constraints

- **Holonomic constraints**
  - $g(q, t) = 0$

- **Non-holonomic constraints**
  - $g(q, q', t) = 0$
  - This is related to the kinematics of robots
  - To accommodate this, the C-space is extended to include the position and its velocity

- **Dynamic constraints**
  - Dynamic equations are represented as $G(q, q', q'') = 0$
  - These constraints are reduced to non-holonomic ones when we use the extended C-space such as the state space
Example of Non-Holonomic Constraints

The path of the car is a curve tangent to its main rotation axis

\[ dx \sin \theta - dy \cos \theta = 0 \]
Computation of Dimension of C-Space

- Suppose that we have a rigid body that can translate and rotate in 2D workspace
  - Start with three points: A, B, C (6D space)

- We have the following (holonomic) constraints
  - Given A, we know the dist to B: \( d(A,B) = |A-B| \)
  - Given A and B, we have similar equations:
    \[ d(A,C) = |A-C|, \quad d(B,C) = |B-C| \]

- Each holonomic constraint reduces one dim.
  - Not for non-holonomic constraint
Example: Rigid Robot in 3-D Workspace

- We can represent the positions and orientations of such robots with matrices (i.e., SO (3) and SE (3))
SO (n) and SE (n)

- **Special orthogonal group, SO(n),** of n x n matrices $R,

$$R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

that satisfy:

$$r_{1i}^2 + r_{2i}^2 + r_{3i}^2 = 1 \text{ for all } i ,$$

$$r_{1i}r_{1j} + r_{2i}r_{2j} + r_{3i}r_{3j} = 0 \text{ for all } i \neq j ,$$

$$\det(R) = +1$$

Refer to the 3D Transformation at the undergraduate computer graphics.

- **Given the orientation matrix** $R$ of SO (n) and the position vector $p$, **special Euclidean group, SE (n),** is defined as:

$$\begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$
Example: Rigid Robot in 3-D Workspace

- $q = \text{(position, orientation)} = (x, y, z, ???)$

- Parametrization of orientations by matrix:
  
  $q = (r_{11}, r_{12}, \ldots, r_{33}, r_{33})$ where $r_{11}, r_{12}, \ldots, r_{33}$ are the elements of rotation matrix

  $R = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \in SO(3)$
Example: Rigid Robot in 3-D Workspace

- Parametrization of orientations by Euler angles: $(\phi, \theta, \psi)$

\[
1 \rightarrow 2 \rightarrow 3 \rightarrow 4
\]
Example: Rigid Robot in 3-D Workspace

- Parametrization of orientations by **unit quaternion**: \( u = (u_1, u_2, u_3, u_4) \) with \( u_1^2 + u_2^2 + u_3^2 + u_4^2 = 1 \).

  - Note \((u_1, u_2, u_3, u_4) = (\cos \theta/2, n_x \sin \theta/2, n_y \sin \theta/2, n_z \sin \theta/2)\) with \( n_x^2 + n_y^2 + n_z^2 = 1 \)

- Compare with representation of orientation in 2-D: 
  \( (u_1, u_2) = (\cos \theta, \sin \theta) \)
Example: Rigid Robot in 3-D Workspace

- Advantage of unit quaternion representation
  - Compact
  - No singularity (no gimbal lock indicating two axises are aligned)
  - Naturally reflect the topology of the space of orientations

- Number of dofs = 6
- Topology: $\mathbb{R}^3 \times SO(3)$
Example: Articulated Robot

- $q = (q_1, q_2, \ldots, q_{2n})$
- Number of dofs = 2
- What is the topology?

An articulated object is a set of rigid bodies connected at the joints.
Class Objectives were:

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics
Next Time….

- Configuration space
  - Definitions and examples
  - Obstacles
  - Paths
  - Metrics
Homework

- **Browse 2**
  ICRA/IROS/RSS/WAFR/TRO/IJRR papers
  - Prepare two summaries and submit at the beginning of every Tue. class, or
  - Submit it online before the Tue. Class

- **Example of a summary (just a paragraph)**
  
  Title: XXX XXXX XXXX
  Conf./Journal Name: ICRA, 2016
  Summary: this paper is about accelerating the performance of collision detection. To achieve its goal, they design a new technique for reordering nodes, since by doing so, they can improve the coherence and thus improve the overall performance.
Homework for Every Class

- Go over the next lecture slides
- Come up with one question on what we have discussed today and submit at the end of the class
  - 1 for typical questions
  - 2 for questions with thoughts or that surprised me

- Write a question at least 4 times before the mid-term exam