
CS482: Radiometry and Rendering Equation

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Course URL:
<http://sglab.kaist.ac.kr/~sungeui/ICG/>



Questions

- **As far as I understood from the slide, we can mix radiosity and ray tracing in hybrid method. Does this mean that we use radiosity for diffuse objects and ray tracing for specular objects? It would be much efficient since you don't have to calculate rays for diffuse objects when the view changes right?**
- **Is there a difference in performance between Jacobi iteration and Gauss-Seidel iteration? Which one is faster or more accurate?**

Announcements

- **Make a project team of 2 or 3 persons for your final project**
 - **Each student has a clear role**
 - **Declare the team at the KLMS by Sep-26; you don't need to define the topic by then**
- **Each student**
 - **Present two papers related to the project**
 - **15 min for each talk, Simple quiz (prepare blank papers)**
- **Each team**
 - **Give a mid-term review presentation for the project**
 - **Give the final project presentation**

Schedule (After Mid-term Exam)

- **Oct. 23 no class, reserved**
- **Oct. 25: Students Presentation I (3 talks per each class)**
- **Oct. 30/Nov-1:**
- **Nov. 6,**
- **Nov 8, 13: Mid-term project presentation**
- **Nov. 15 : SP II (3 talks per each class)**
- **Nov. 20, 22**
- **Nov. 27**
- **Nov. 29: no class (no class due to undergraduate interview)**
- **Dec. 4/6: Final project presentation**
- **Dec. 11, 13 Reserved (final exam week; no exam for us)**

Deadlines

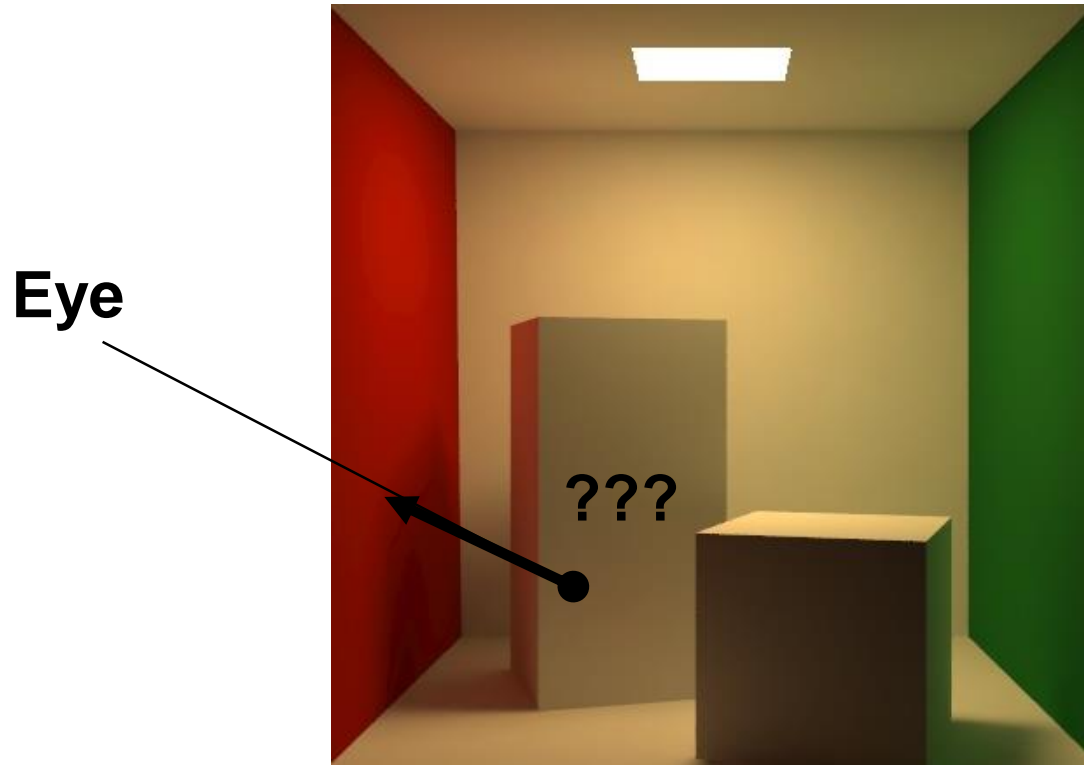
- **Declare project team members**
 - **By 9/26 at KLMS**
 - **Confirm schedules of paper talks and project talks at 9/27**

- **Declare two papers for student presentations**
 - **by 10/10 at KLMS**
 - **Discuss them at the class of 10/11**
 - **Choose graphics papers from 2019 ~ published on top-tier conf. (SIGGRAPH, CVPR, etc.)**

Class Objectives (Ch. 12 and 13)

- **Know terms of:**
 - **Hemispherical coordinates and integration**
 - **Various radiometric quantities (e.g., radiance)**
 - **Basic material function, BRDF**
 - **Understand the rendering equation**

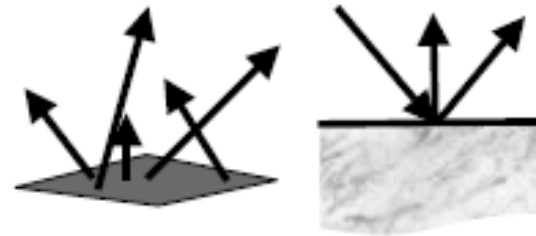
Motivation



Light and Material Interactions

- **Physics of light**
- **Radiometry**
- **Material properties**

- **Rendering equation**



From kavita's slides

Models of Light

- **Quantum optics**
 - **Fundamental model of the light**
 - **Explain the dual wave-particle nature of light**
- **Wave model**
 - **Simplified quantum optics**
 - **Explains diffraction, interference, and polarization**
- **Geometric optics**
 - **Most commonly used model in CG**
 - **Size of objects \gg wavelength of light**
 - **Light is emitted, reflected, and transmitted**

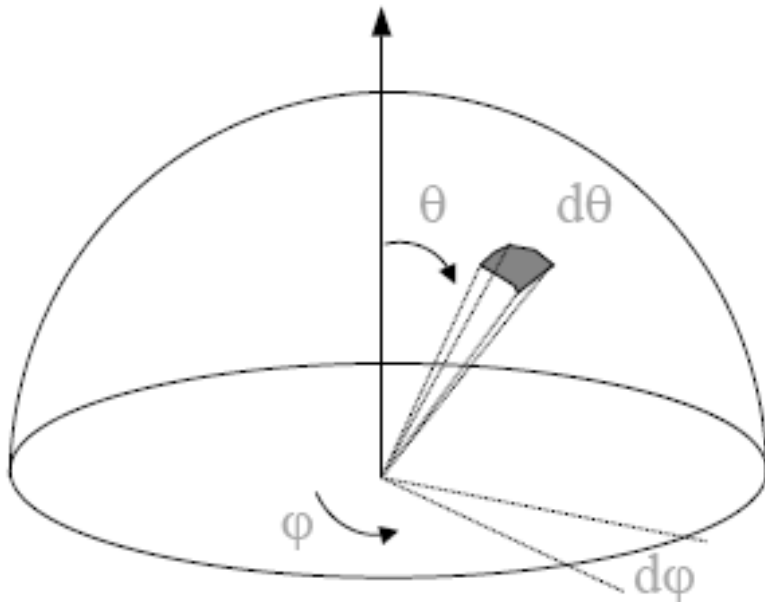


Radiometry and Photometry

- **Photometry**
 - **Quantify the perception of light energy**
- **Radiometry**
 - **Measurement of light energy: critical component for photo-realistic rendering**
 - **Light energy flows through space, and varies with time, position, and direction**
 - **Radiometric quantities: densities of energy at particular places in time, space, and direction**
 - **Briefly discussed here; refer to my book**

Hemispheres

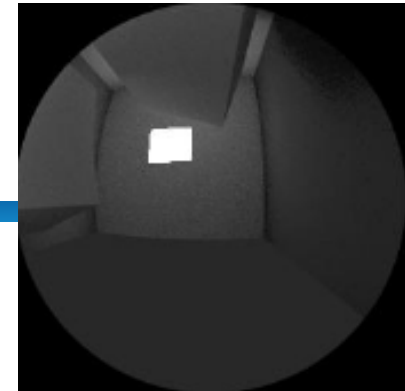
- Hemisphere
 - Two-dimensional surfaces
- Direction
 - Point on (unit) sphere



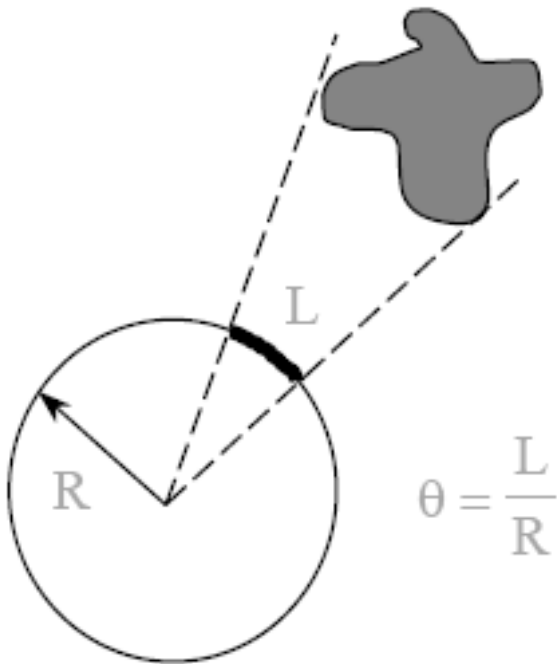
$$\theta \in [0, \frac{\pi}{2}]$$
$$\varphi \in [0, 2\pi]$$

From kavita's slides

Solid Angles



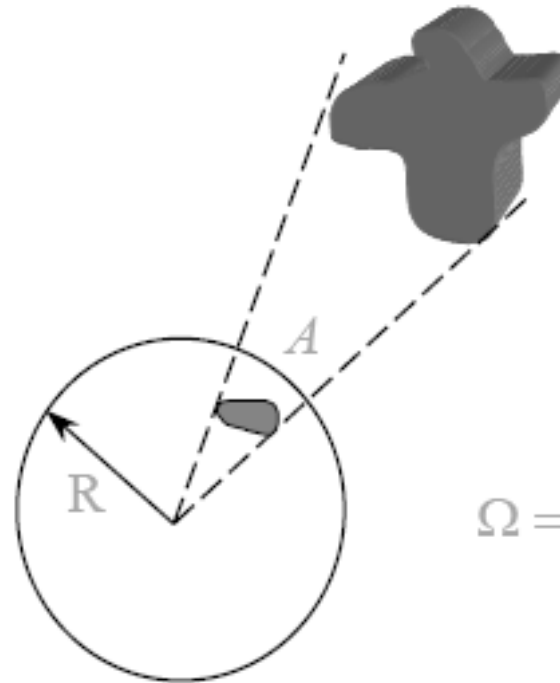
2D



$$\theta = \frac{L}{R}$$

Full circle
= 2π radians

3D



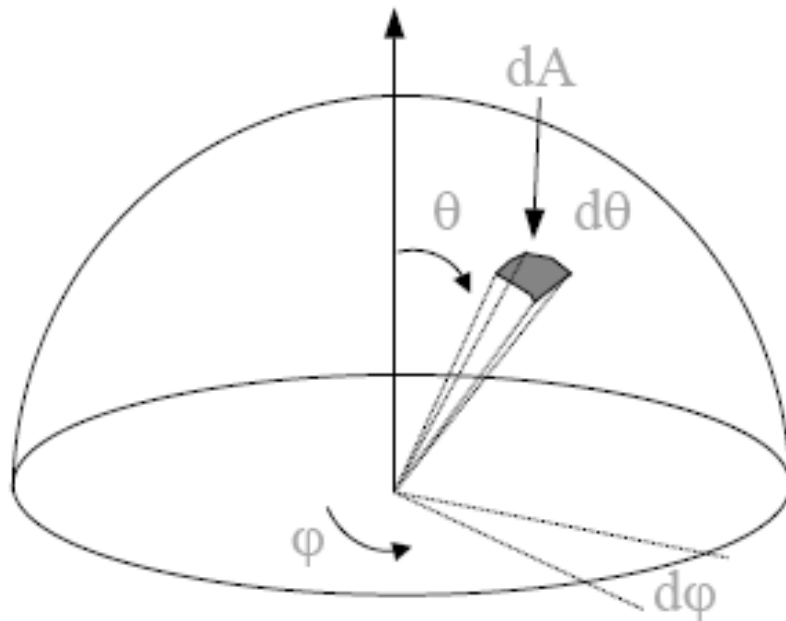
$$\Omega = \frac{A}{R^2}$$

Full sphere
= 4π steradians

View on the
hemisphere

Hemispherical Coordinates

- Direction, \odot
 - Point on (unit) sphere

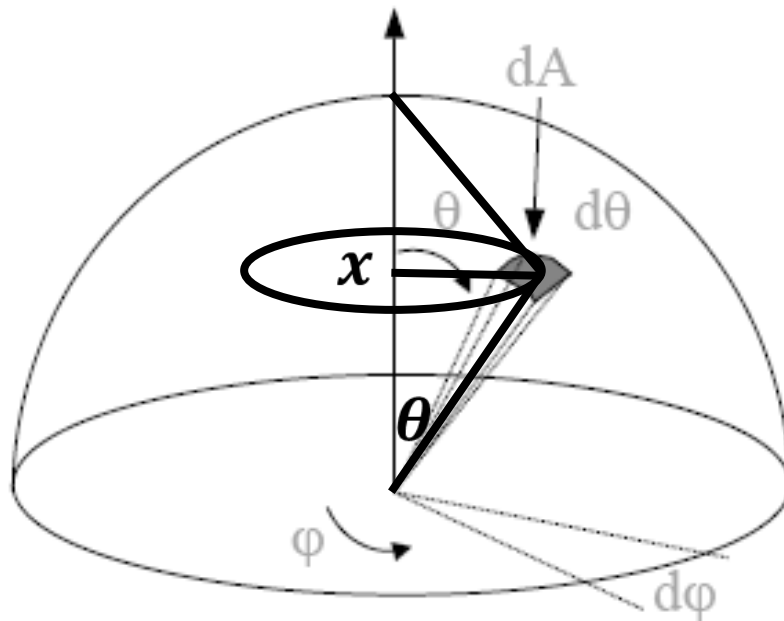


$$dA = (r \sin \theta d\phi)(r d\theta)$$

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Hemispherical Coordinates

- Direction, \ominus
 - Point on (unit) sphere



$$\sin \theta = \frac{x}{r},$$
$$x = r \sin \theta$$

$$dA = (r \sin \theta d\varphi)(r d\theta)$$

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Hemispherical Coordinates

- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

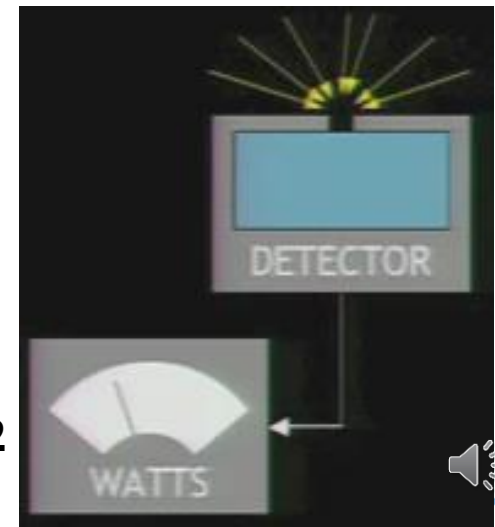
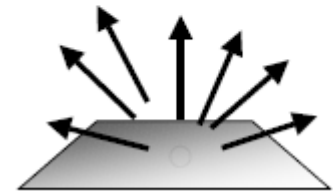
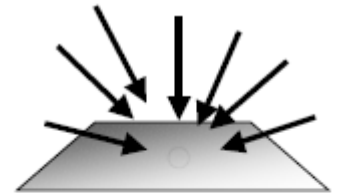
Hemispherical Integration

- Area of hemisphere:

$$\begin{aligned}\int_{\Omega_x} d\omega &= \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta \\ &= \int_0^{2\pi} d\varphi [-\cos\theta]_0^{\pi/2} \\ &= \int_0^{2\pi} d\varphi \\ &= 2\pi\end{aligned}$$

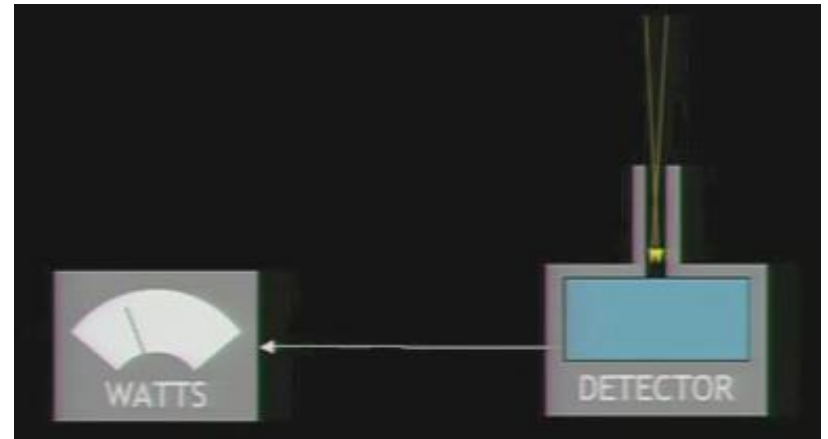
Irradiance

- **Incident radiant power per unit area (dP/dA)**
 - **Area density of power**
- **Symbol: E , unit: W/m^2**
 - **Area power density exiting a surface is called radiance exitance (M) or radiosity (B)**
- **For example**
 - **A light source emitting 100 W of area $0.1 m^2$**
 - **Its radiant exitance is $1000 W/m^2$**



Radiance

- **Radiant power at x in direction θ**
 - $L(x \rightarrow \Theta)$: 5D function
 - Per unit area
 - Per unit solid angle

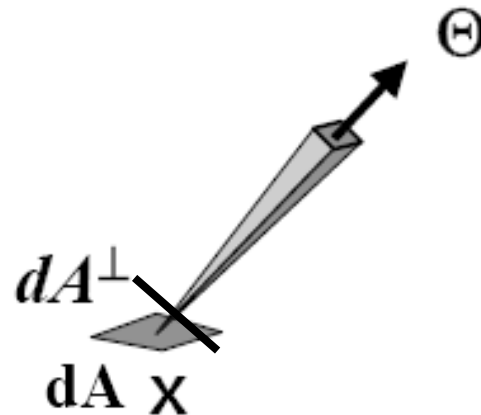


- **Important quantity for rendering**

Radiance

- **Radiant power at x in direction Θ**
 - $L(x \rightarrow \Theta)$: 5D function
 - Per unit area
 - Per unit solid angle

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

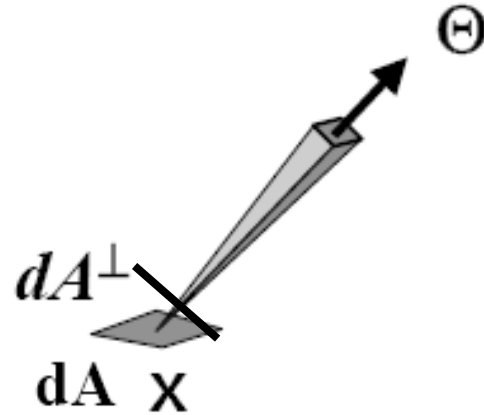


- **Units: Watt / (m² sr)**
- **Irradiance per unit solid angle**
- **2nd derivative of P**
- **Most commonly used term**

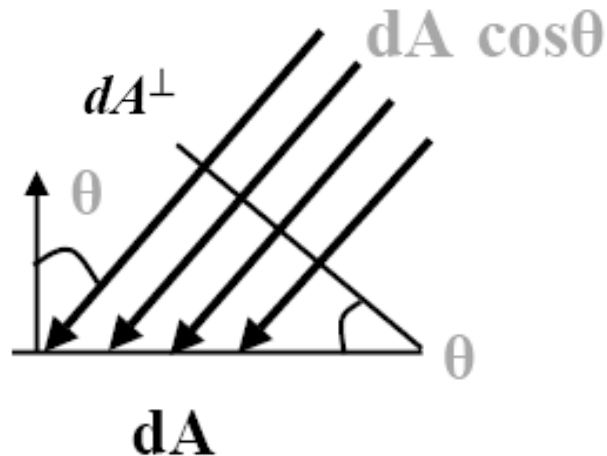
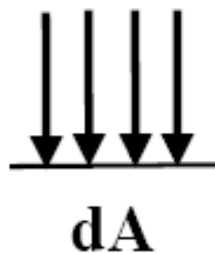
Radiance: Projected Area

$$L(x \rightarrow \Theta) = \frac{d^2 P}{dA^\perp d\omega_\Theta}$$

$$= \frac{d^2 P}{d\omega_\Theta dA \cos \theta}$$

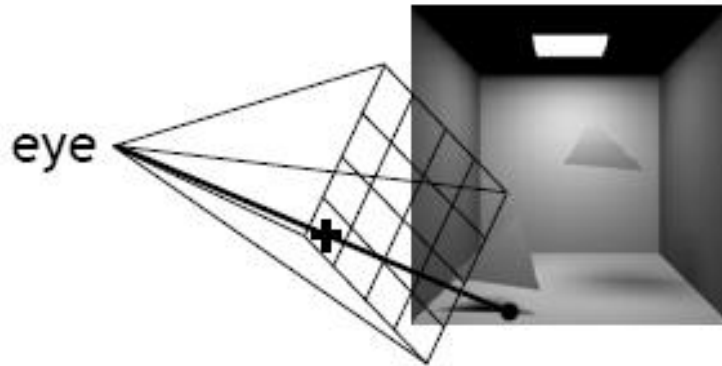


- Why per unit projected surface area



Sensitivity to Radiance

- Responses of sensors (camera, human eye) is proportional to radiance

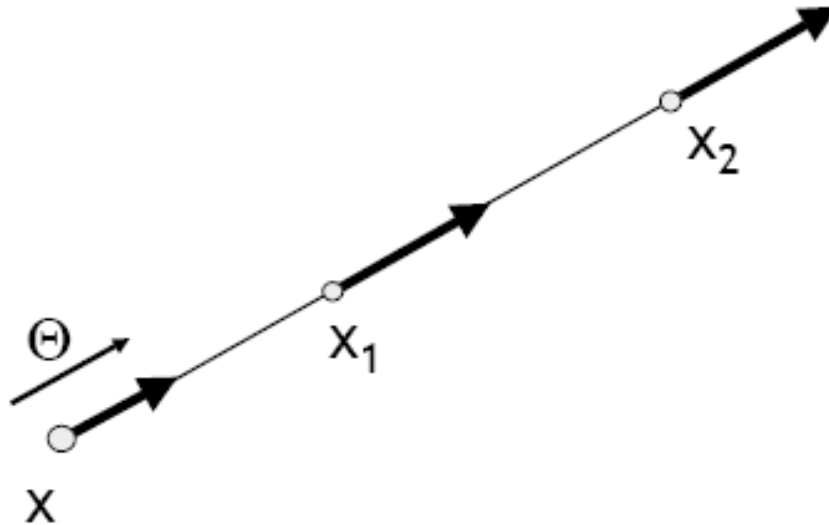


From kavita's slides

- Pixel values in image proportional to radiance received from that direction

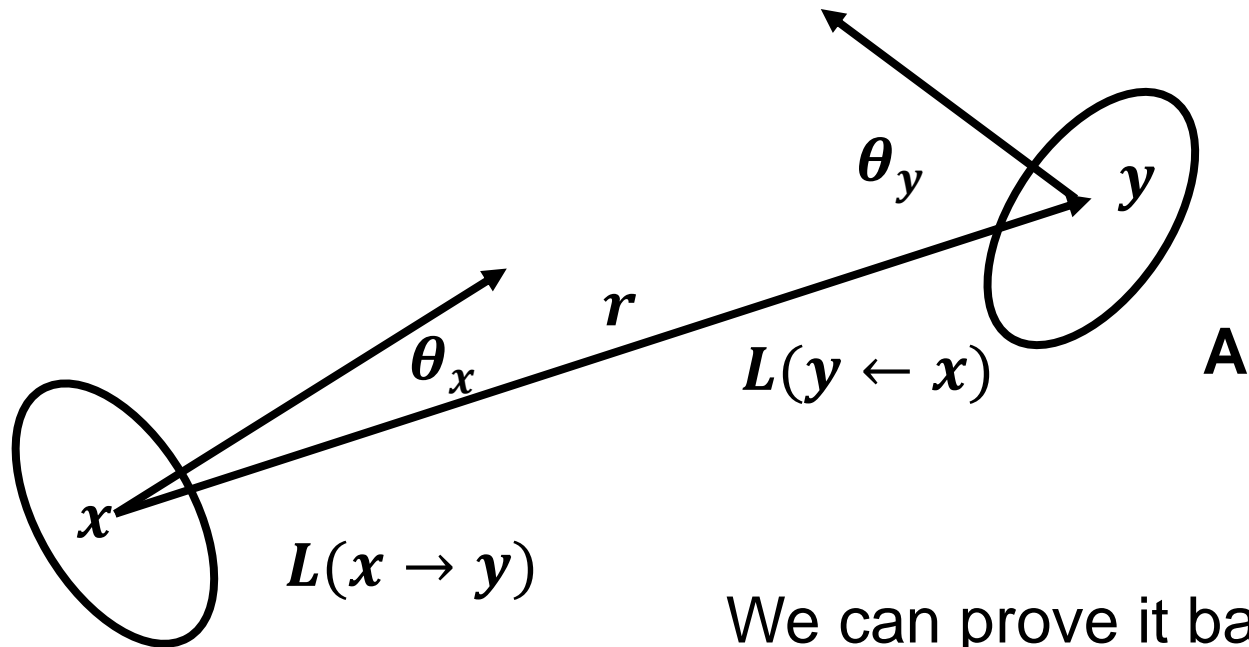
Properties of Radiance

- Invariant along a straight line (in vacuum)



From kavita's slides

Invariance of Radiance



We can prove it based on the assumption the conservation of energy.

Relationships

- Radiance is the fundamental quantity

$$L(x \rightarrow \Theta) = \frac{d^2P}{dA^\perp d\omega_\Theta}$$

- Power:

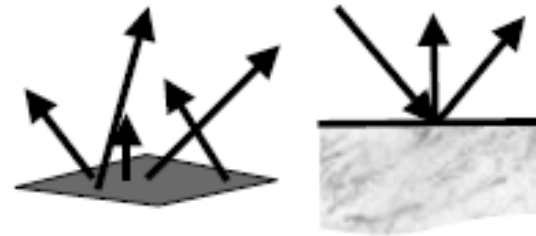
$$P = \int_{\substack{\text{Area} \\ \text{Solid} \\ \text{Angle}}} \int L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta \cdot dA$$

- Radiosity:

$$B = \int_{\substack{\text{Solid} \\ \text{Angle}}} L(x \rightarrow \Theta) \cdot \cos \theta \cdot d\omega_\Theta$$

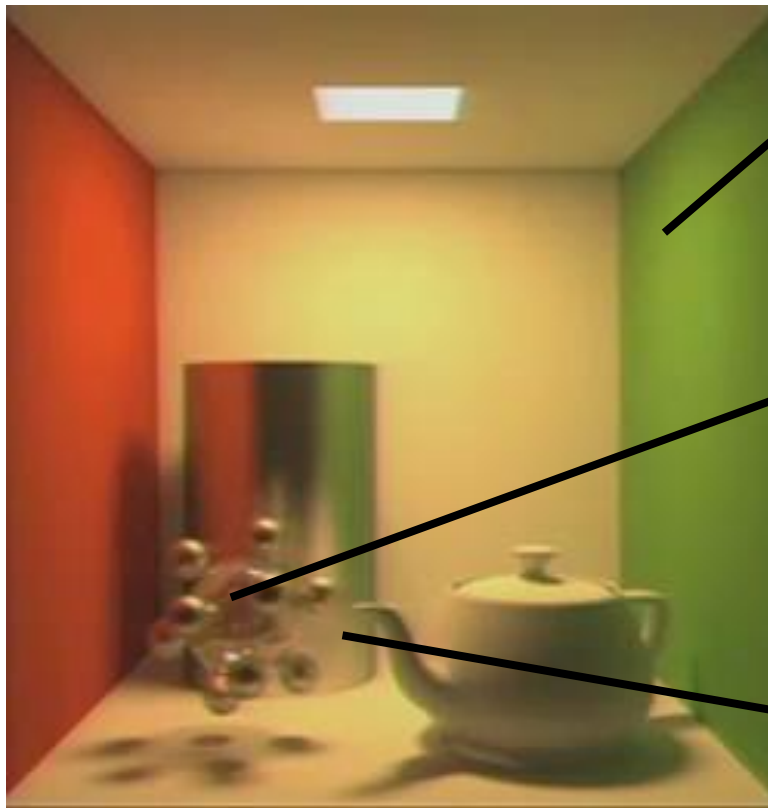
Light and Material Interactions

- Physics of light
- Radiometry
- **Material properties**
- Rendering equation



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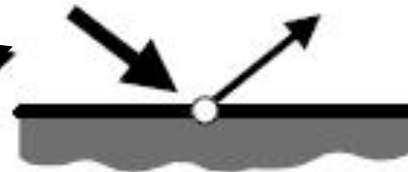
Materials



From kavita's slides



**Ideal diffuse
(Lambertian)**

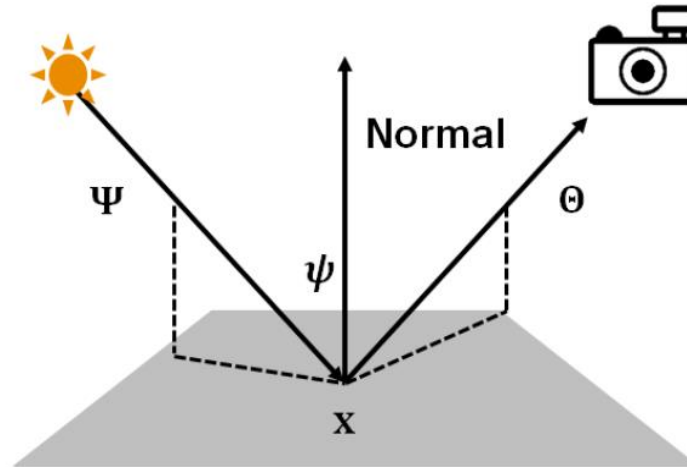


Ideal specular



Glossy

Bidirectional Reflectance Distribution Function (BRDF)

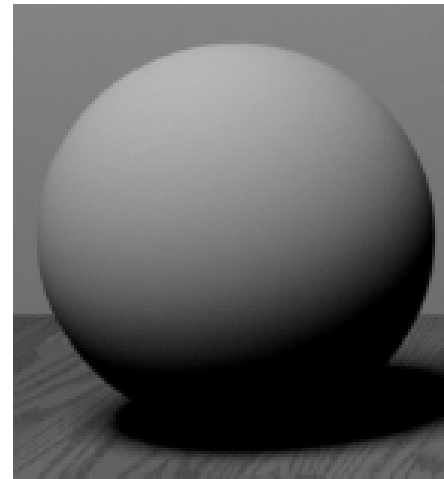


$$f_r(x, \Psi \rightarrow \Theta) = \frac{dL(x \rightarrow \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \rightarrow \Theta)}{L(x \leftarrow \Psi) \cos \psi d\omega_\Psi}$$

BRDF special case: ideal diffuse

Pure Lambertian

$$f_r(x, \Psi \rightarrow \Theta) = \frac{\rho_d}{\pi}$$

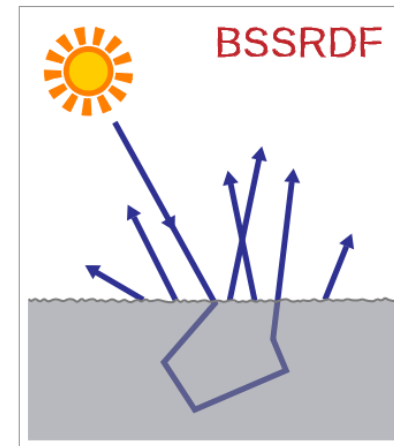
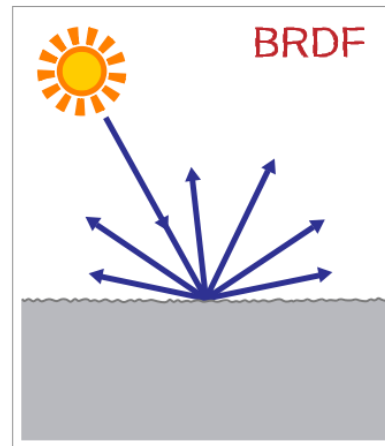
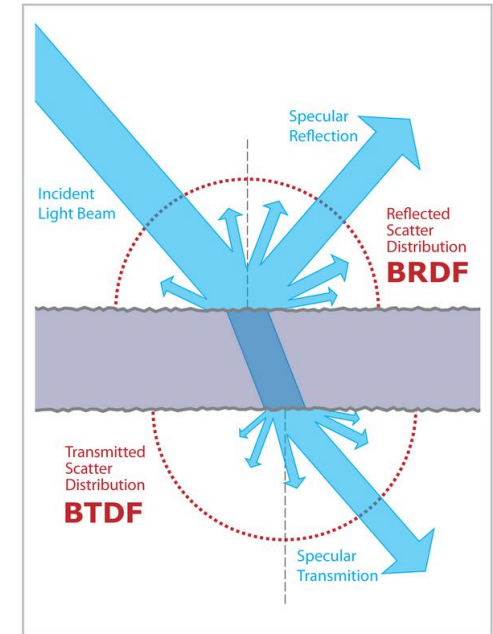


$$\rho_d = \frac{\text{Energy}_{out}}{\text{Energy}_{in}} \quad 0 \leq \rho_d \leq 1$$



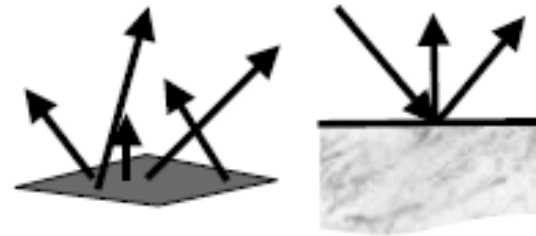
Other Distribution Functions: BxDF

- **BSDF (S: Scattering)**
 - The general form combining BRDF + BTDF (T: Transmittance)
- **BSSRDF (SS: Surface Scattering)**
 - Handle subsurface scattering



Light and Material Interactions

- Physics of light
- Radiometry
- Material properties
- **Rendering equation**



From kavita's slides

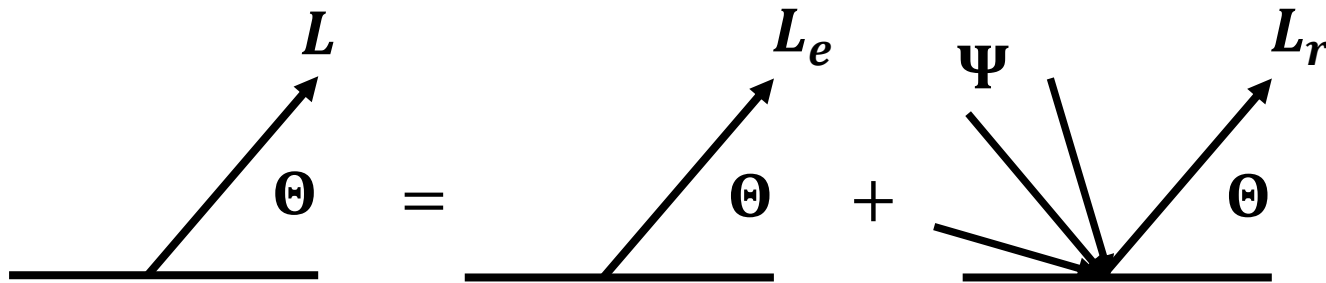
Light Transport

- **Goal**
 - **Describe steady-state radiance distribution in the scene**
- **Assumptions**
 - **Geometric optics**
 - **Achieves steady state instantaneously**

Rendering Equation

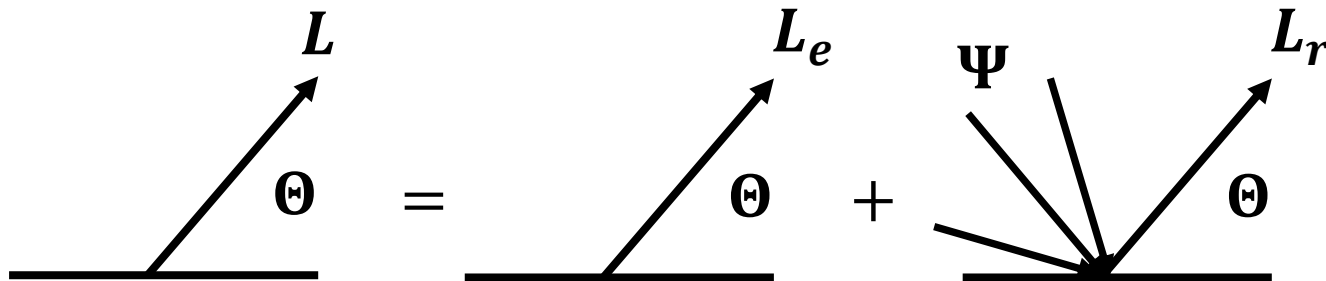
- **Describes energy transport in the scene**
- **Input**
 - **Light sources**
 - **Surface geometry**
 - **Reflectance characteristics of surfaces**
- **Output**
 - **Value of radiances at all surface points in all directions**

Rendering Equation



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + L_r(x \rightarrow \Theta)$$

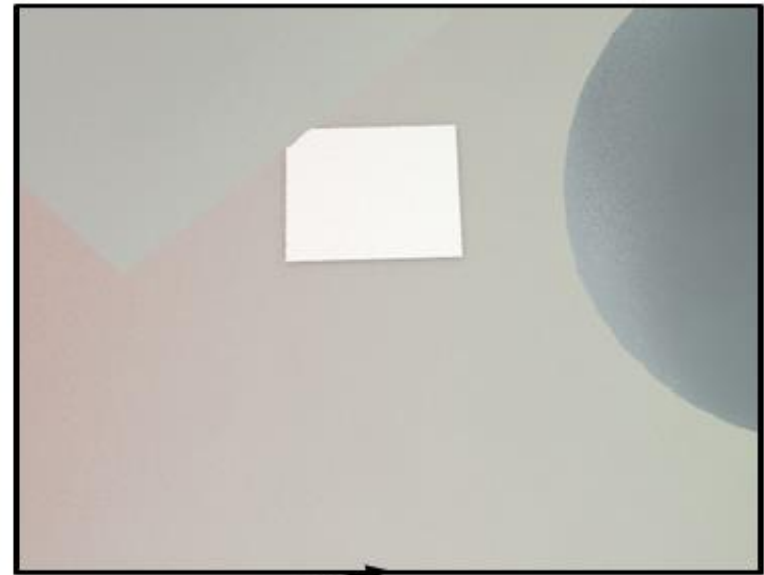
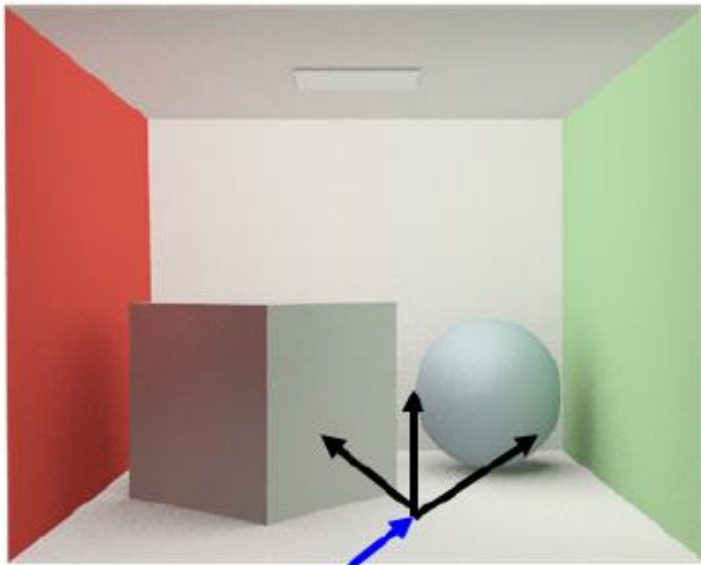
Rendering Equation



$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x dw_{\Psi},$$

- Applicable to all wave lengths

Rendering Equation

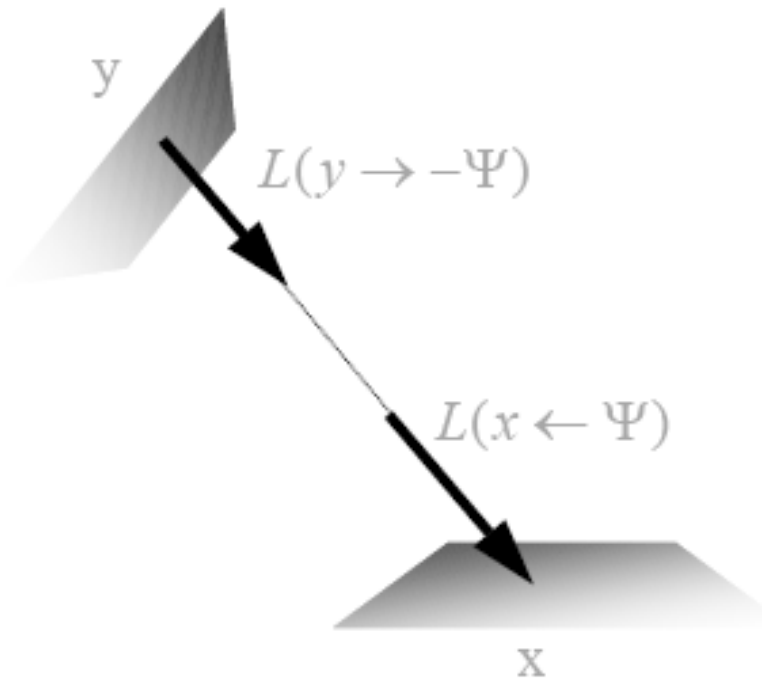


Incoming radiance on the hemisphere

$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x dw_{\Psi}$$

Rendering Equation: Area Formulation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$



Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ ?

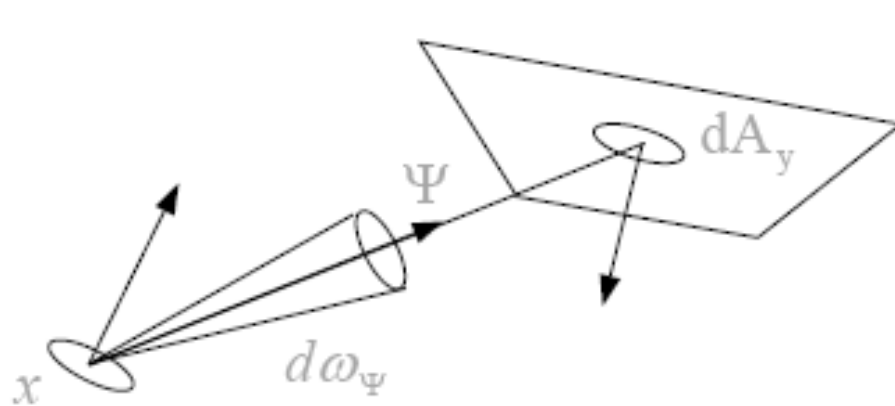
$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$



Rendering Equation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos \theta_y}{r_{xy}^2}$$

Rendering Equation: Visible Surfaces

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos \theta_x \cdot d\omega_\Psi$$

Coordinate transform



$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \rightarrow -\Psi) \cos \theta_x \cdot \frac{\cos \theta_y}{r_{xy}^2} \cdot dA_y$$



$$y = vp(x, \Psi)$$



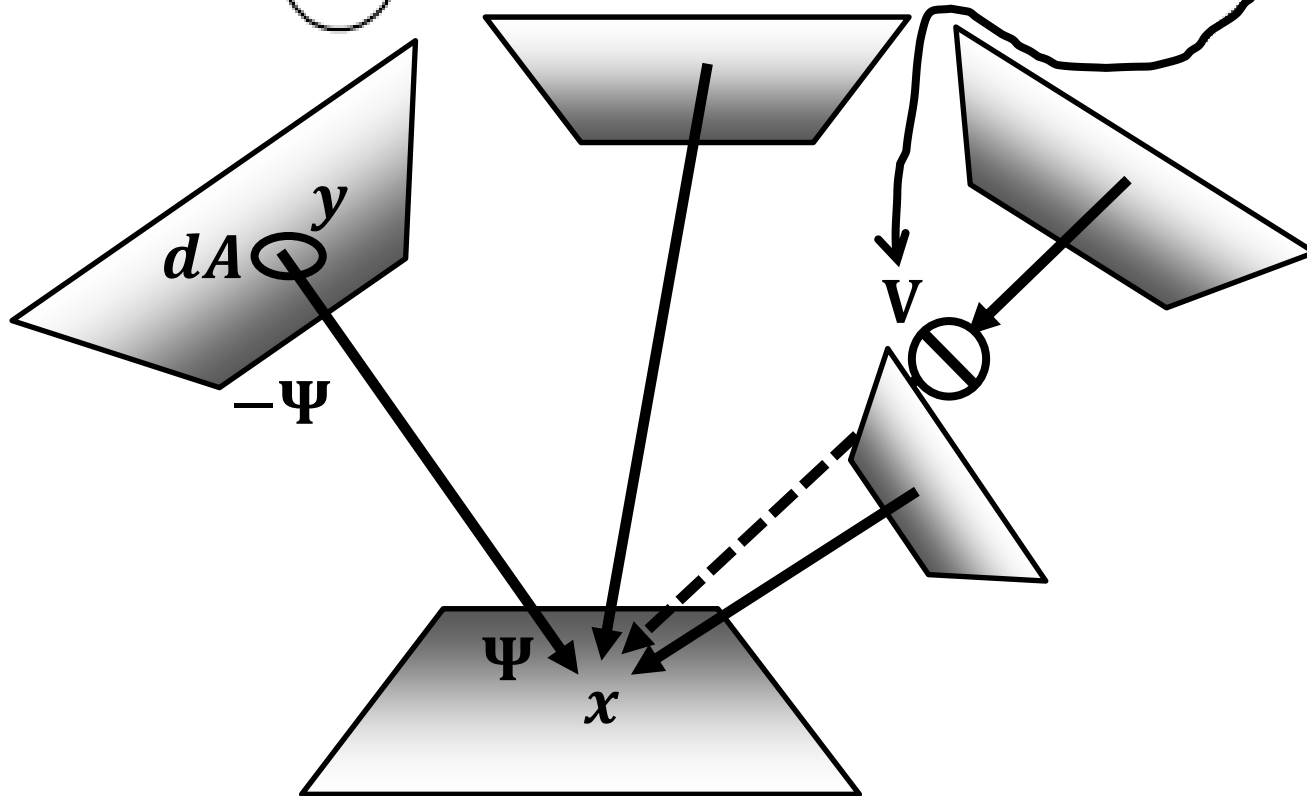
Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function



Rendering Equation: All Surfaces

$$L(x \rightarrow \Theta) = L_e(\dots) + \int_A f_r(\dots) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos \theta_x \cdot \cos \theta_y}{r_{xy}^2} \cdot V(x, y) dA_y$$



Two Forms of the Rendering Equation

- **Hemisphere integration**

$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi}$$

- **Area integration (used as the form factor for radiosity)**

$$L_r(x \rightarrow \Theta) = \int_A L(y \rightarrow -\Psi) f_r(x, \Psi \rightarrow \Theta) \frac{\cos \theta_x \cos \theta_y}{r_{xy}^2} V(x, y) dA,$$

Class Objectives (Ch. 12 & 13) were:

- **Know terms of:**
 - **Hemispherical coordinates and integration**
 - **Various radiometric quantities (e.g., radiance)**
 - **Basic material function, BRDF**
 - **Understand the rendering equation**

Any Questions?

- **Submit four times in Sep./Oct.**
- **Come up with one question on what we have discussed in the class and submit at the end of the class**
 - **1 for typical questions**
 - **2 for questions that have some thoughts or surprise me**

Next Time

- **Monte Carlo rendering methods**

Homework

- **Go over the next lecture slides before the class**
- **Watch two videos or go over papers, and submit your summaries every Mon. class**
 - **Just one paragraph for each summary**

Example:

Title: XXX XXXX XXXX

Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.