Integrating Clipped Spherical Harmonics Expansions

Laurent Belcour at al. 2018 ACM TOG

Presenter: 조인영 - In Young Cho

Analytic Solution RETURNS

- Real-time rendering, AGAIN!
    - Analytic solution of the rendering equation

- My first paper presentation: LTC
- My project title: LTC for shadow
- This paper presentation: *Spherical Harmonics*

Motivation of This Paper

- Let's approximate BRDFs
- Let's approximate *randomly shaped* BRDFs!!

- Also, the integration should be easy

Contribution of This Paper

1. Found a scheme to integrate
   a. not only simple BRDFs
   b. but *ANY* arbitrary spherical functions
   c. efficiently

2. Apply the scheme to
   a. surface rendering
   b. importance sampling
   c. control variate (reduce MCRT's noise)

Need Visual Aids?

https://belcour.github.io/blog/slides/2018-integration-sh/slides.html#/18

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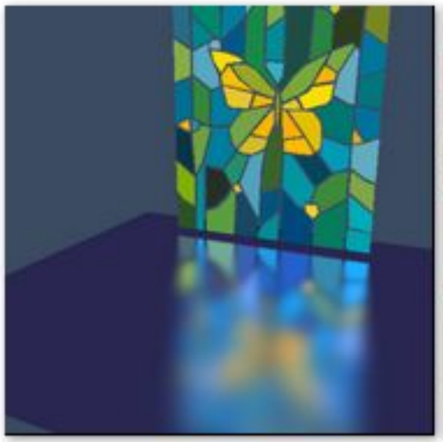
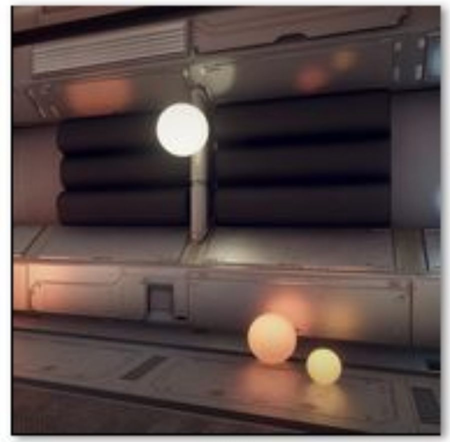
A Few Existing Methods

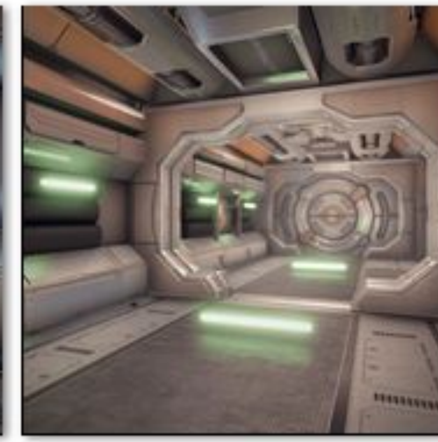


Arvo1995

Heitz2016



Dupuy2017

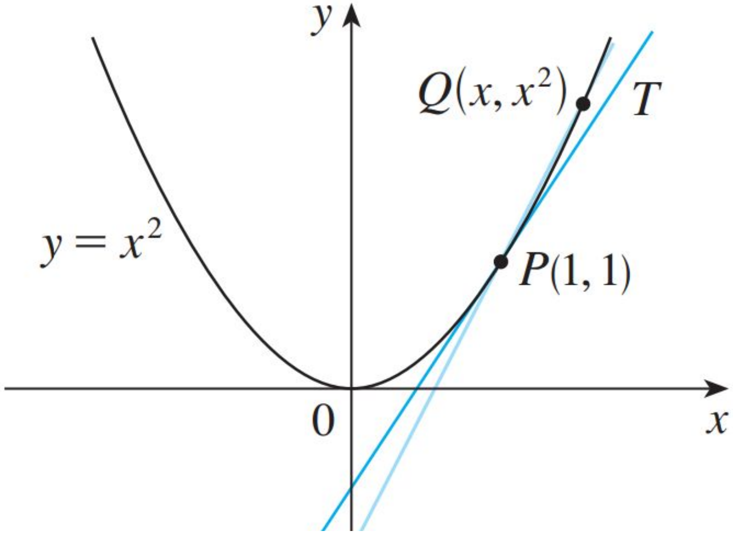


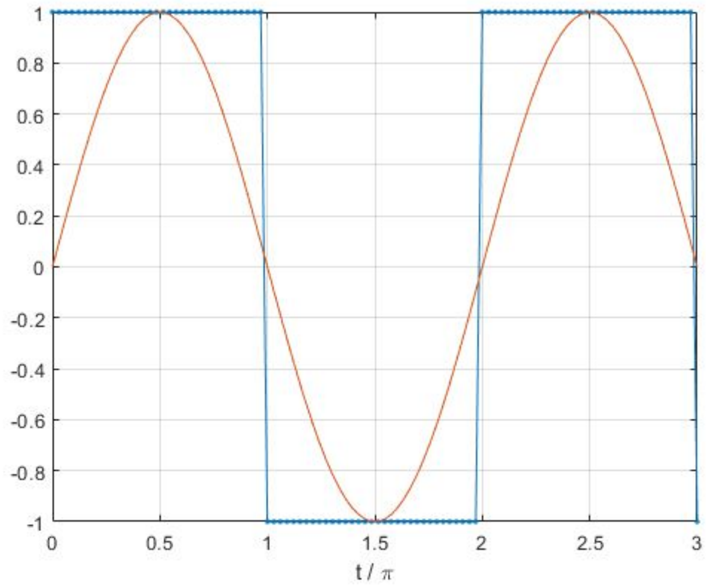
Heitz2017

- But approximate the BRDF with only one & simple function
  - Heitz LTC paper, use LTC alone
- Hard to get good approximations of complex BRDFs

Single Function Approximations

imgur.com & MATLAB



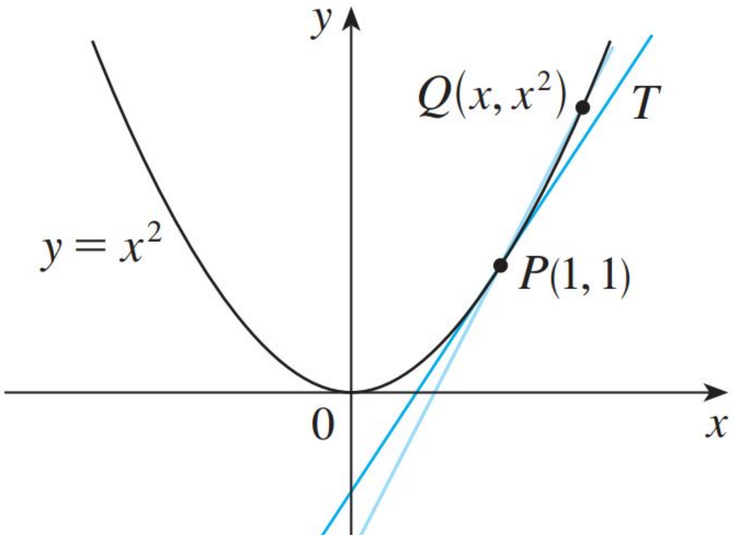


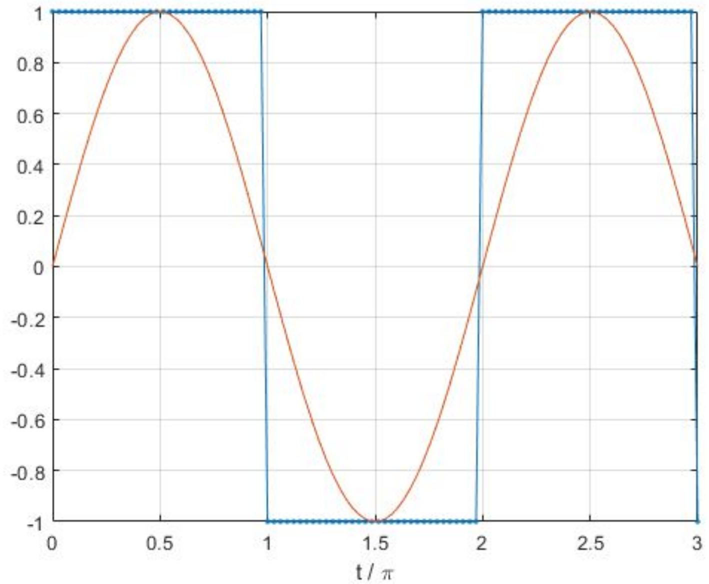
$$y = ax + b$$

$$y = a\cos(bx)$$

Single Function Approximations

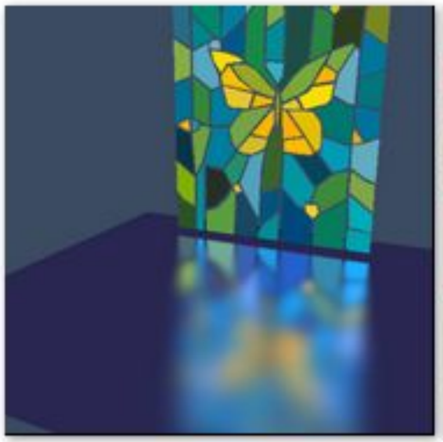
imgur.com &
MATLAB





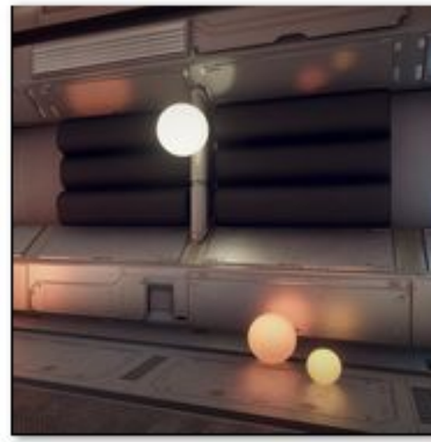
- Works well in very narrow ranges
- Not works in broad range
- Not works for sophisticated functions

A Few Existing Methods

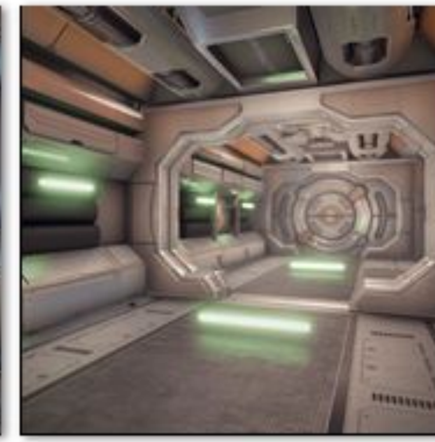


Arvo1995

Heitz2016



Dupuy2017



Heitz2017

- **Hard to get good approximations of complex BRDFs**

Can We Do Better?

- Good approximation of arbitrarily, sophisticated shaped BRDFs
  - Efficient computation of the integrals

Key Concept (1): Spherical Harmonics

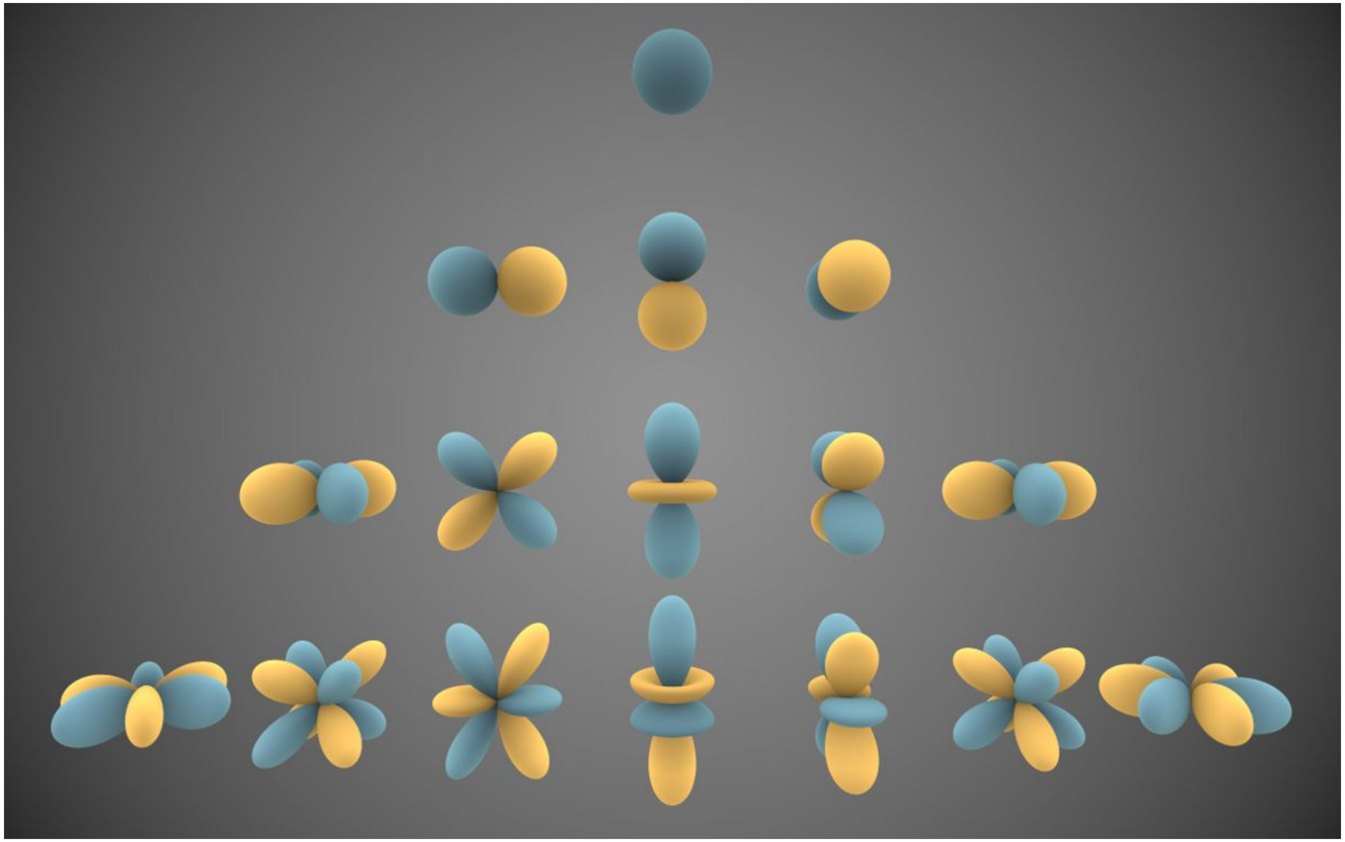
Key Idea: Series of Functions

- **Taylor series**
  - $f(x) \approx P_n(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots$
  - **rather than a single linear line**

- **Fourier series**
  - $f(x) \approx f_n(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) \ldots$
  - **rather than a single cosine**

- **Add the more complex terms behind, the better**

Spherical Harmonics



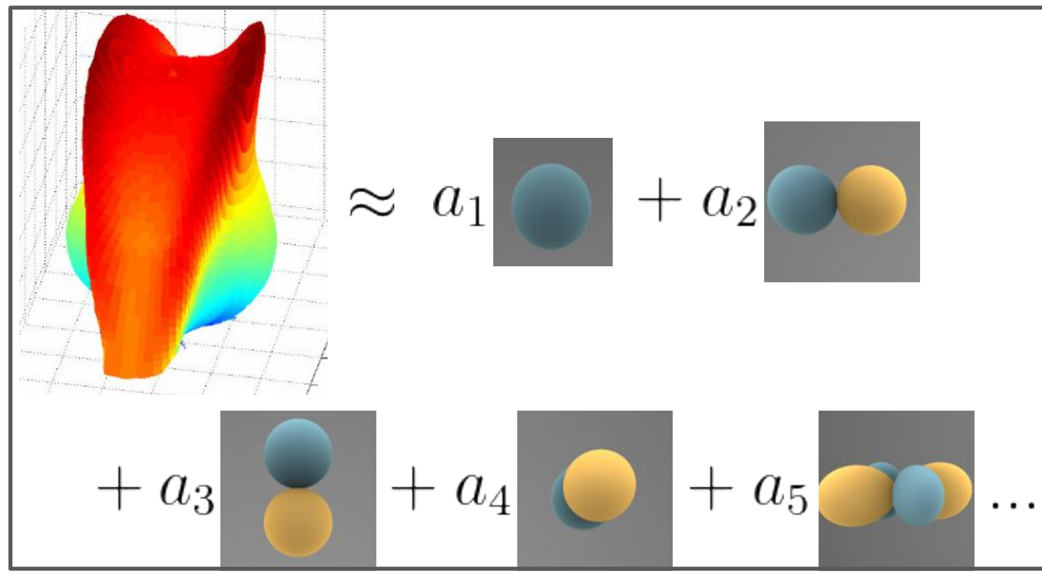
https://en.wikipedia.org/wiki/File:Spherical_Harmonics.png

Simple shape

Complex
Finer
Oscilicate

Spherical Harmonics Expansion

- **Taylor:** $f(x) \approx P_n(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \ldots$
- **Fourier:** $f(x) \approx f_n(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) +$
- **SH:**



- **Any spherical function can be decomposed in an infinite SH expansion!**

BRDF, Silk img by Ngan2005

In Fancy Words….

Spherical Harmonics are an orthonormal basis of functions defined on the unit sphere.

- Even if you cannot get it, it's okay~~!

Approximate BRDF Integration

$$f(\omega) = \sum f_{l,m} \underline{y_l^m(\omega)}$$

Spherical Harmonic

$$\int_P f(\omega) d\omega = \sum f_{l,m} \int_P y_l^m(\omega) d\omega$$

- **Integral of BRDF is nothing but sum of integral of Spherical Harmonics**

Key Concept (2):
So, How to Integrate SHs??
<Powers of Cosine>

Spherical Harmonics

- **Sum of powers of cosine**

$$SH = Cosine + Cosine^2 + \cdots + Cosine^n$$

Power Cosine Integration

$$\int (\cos x)^n dx = \frac{(\cos x)^{n-1} sinx}{n} + \frac{n-1}{n} \int (\cos x)^{n-2} dx$$

- Recursive integration
- If we store previous terms, we can exploit them to compute the integral of next power cosine

$$SH = Cosine + Cosine^2 + \cdots + Cosine^n$$

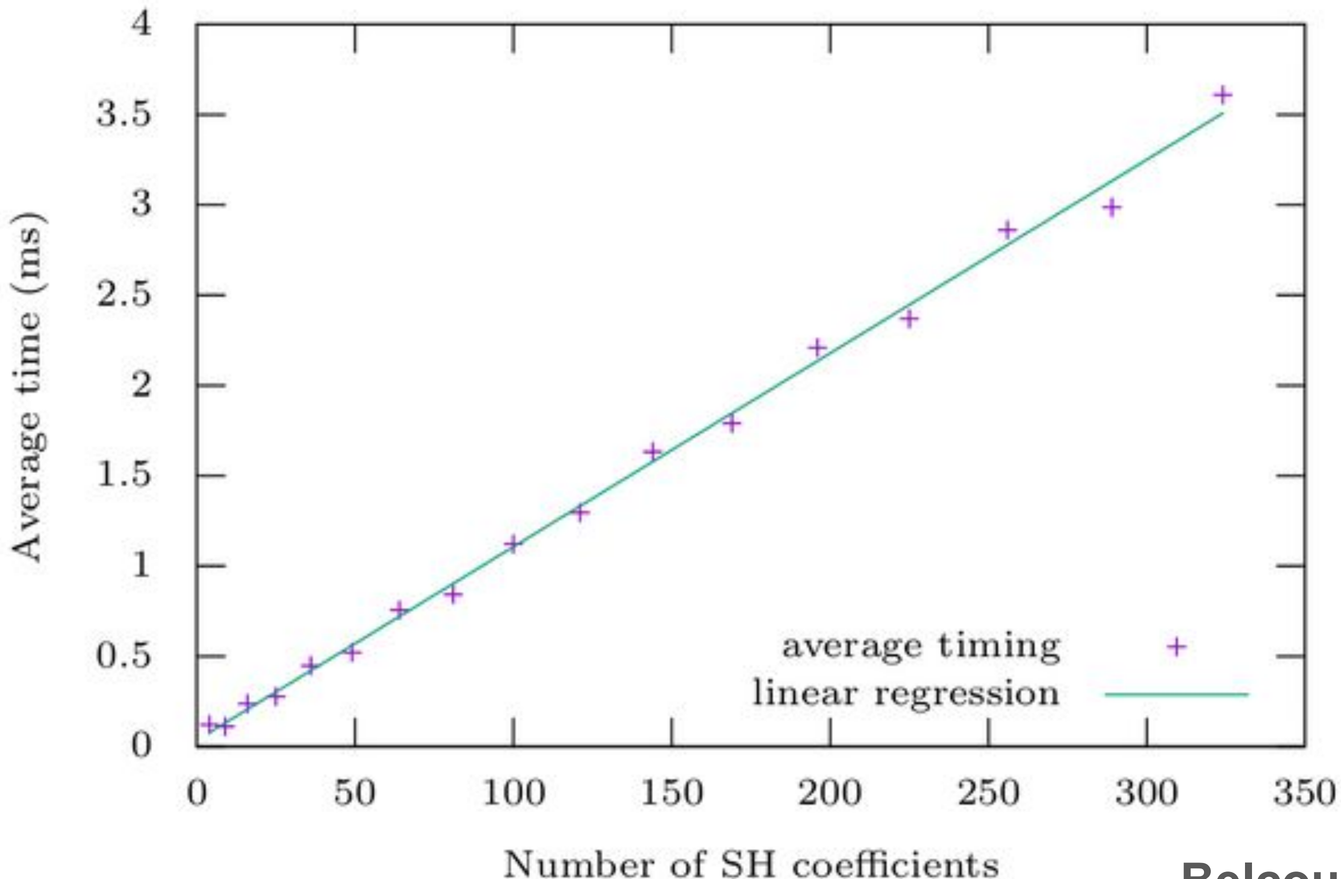
Substitution Property of Cosine

- Recursive Integration [Arvo1995]
  - over a spherical polygon
  - plays an important role in computational complexity drop

- Overall,
  - Integral of BRDF → Sum of Integral of SHs
  - Integral of SHs → Sum of Integral of PowCos
  - Exploit recursive property of powers of cosines

Algorithmic Complexity

- **Linear w.r.t. number of SH coefficients**



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At the End of Our Journey

- Spherical Harmonics expansion
  - How to approximate BRDFs?
  - Taylor or Fourier series for spherical functions
- Power cosine integration
  - How to compute the integral of SH?
  - Thanks to math, cheap to compute
- SH approximates any function on sphere

Quiz

Quiz

1. **What** is the **benefit** of Spherical Harmonics approximation? (against existing analytical solutions)

2. **Which property** of powers of cosine plays an important role in computational **complexity**?

Supplementary Slides

Spherical Harmonics Expansion

$$y_l^m(\theta,\phi) = K_l^m \begin{cases} \cos(m\phi)\ P_l^m(\cos\theta), & m \geq 0 \\ \sin(|m|\phi)\ P_l^{|m|}(\cos\theta), & m < 0 \end{cases}$$

$$I = \sum_{l,m} f_l^m \int_{\mathcal{P}} y_l^m(\vec{\boldsymbol{\omega}})\, \mathrm{d}\vec{\boldsymbol{\omega}}\,.$$

$$I = \sum_{l,m} \sum_{\overline{m},d} \sum_{k} f_l^m\, \alpha_{l,m}^{\overline{m}}\, p_k \int_{\mathcal{P}} \cos^k \theta_d\, \mathrm{d}\omega$$

Arvo, 1995

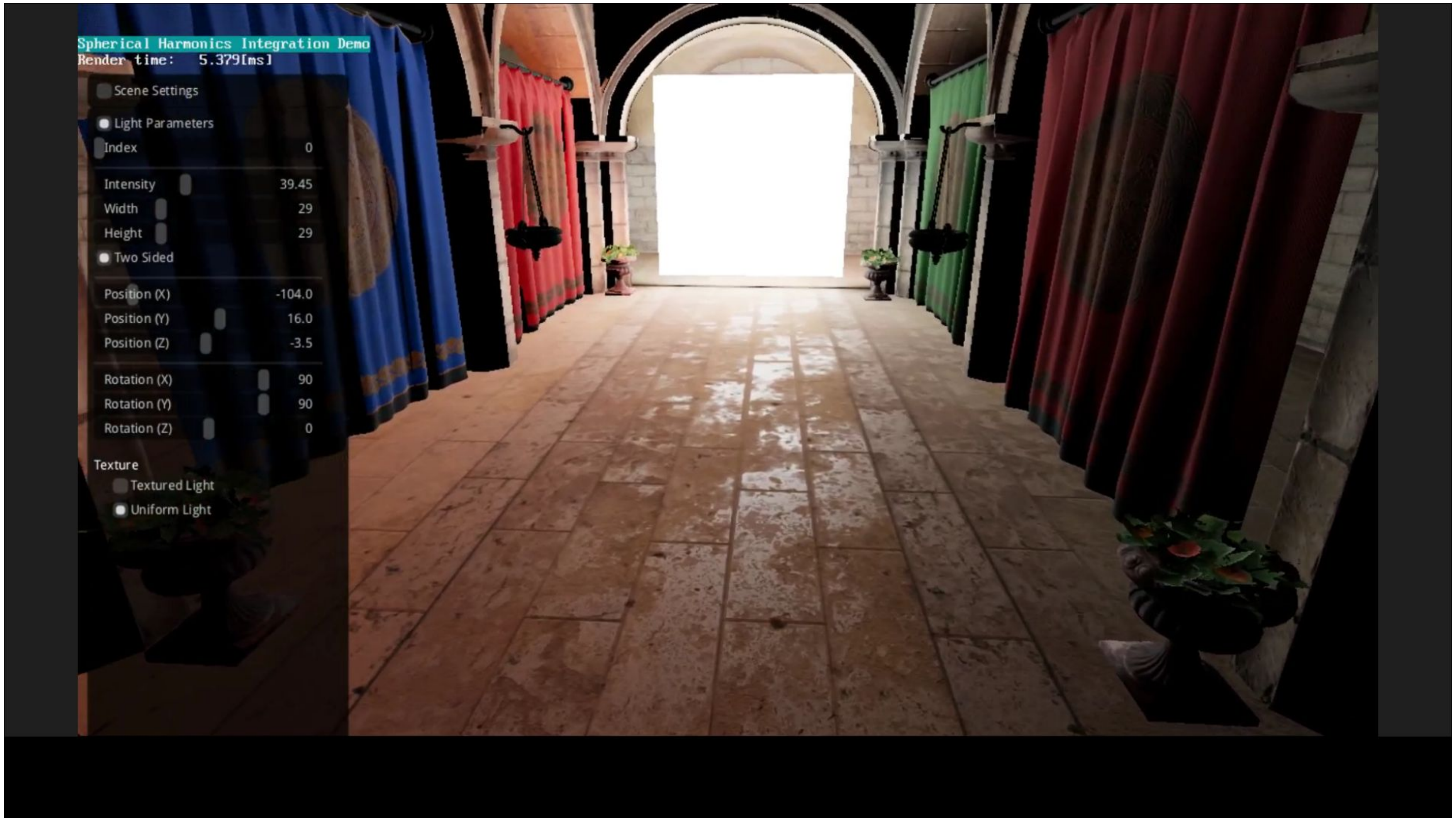
$$\bar{\tau}^n(A, \mathbf{w}) \equiv \int_A (\mathbf{w} \cdot \mathbf{u})^n \; d\sigma(\mathbf{u}).$$

$$\begin{aligned}(n+1)\;\bar{\tau}^n \quad &= \quad (n-1)\;(\mathbf{w} \cdot \mathbf{w})\;\bar{\tau}^{n-2} \\ &- \quad \int_{\partial A} (\mathbf{w} \cdot \mathbf{u})^{n-1}\;\mathbf{w} \cdot \mathbf{n}\, ds,\end{aligned}$$

$$\int (\cos x)^n dx = \frac{(\cos x)^{n-1} sinx}{n} + \frac{n-1}{n} \int (\cos x)^{n-2} dx$$

Applications

- **Sponza scene**



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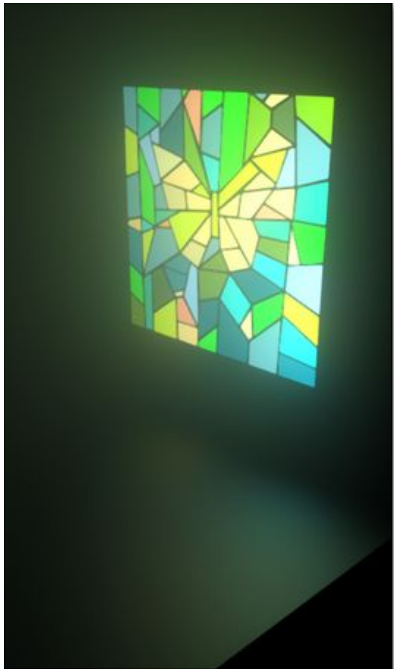
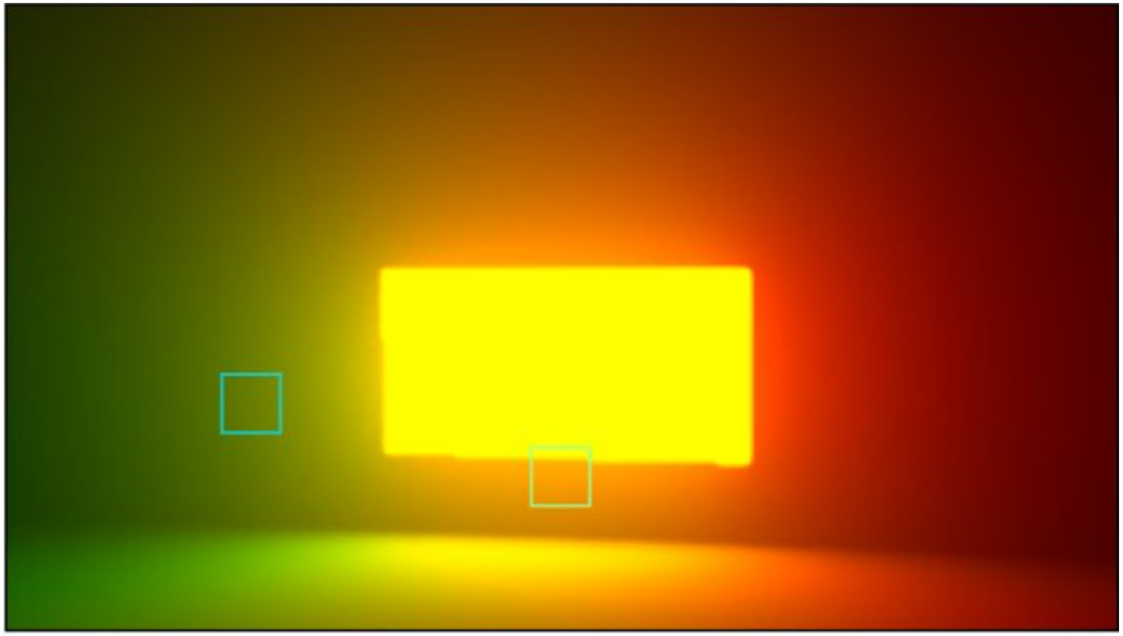
Applications

- **Fog rendering**





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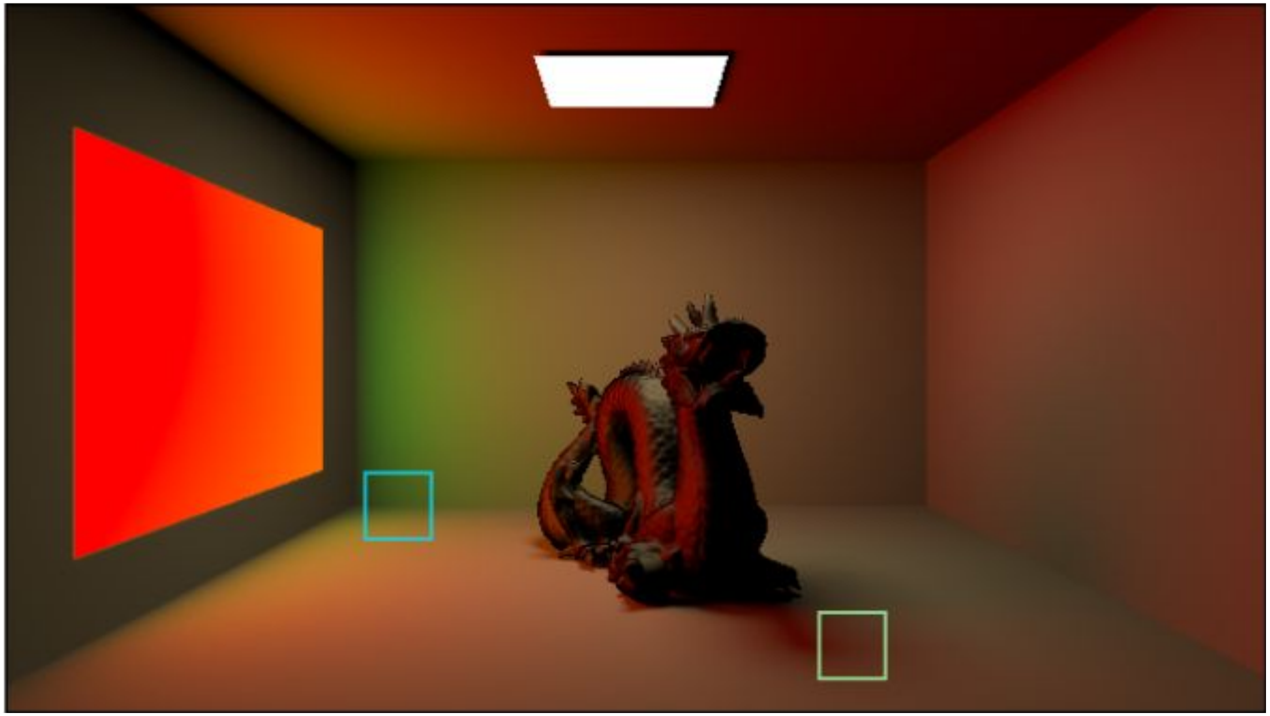
- **Details are in the paper**

Applications

- **Shadow (with MC ray tracing)**



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