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# CS482: Monte Carlo Ray Tracing:

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**KAIST**

The KAIST logo consists of the letters 'KAIST' in a bold, blue, sans-serif font. Below the text is a light blue, horizontal oval shape that serves as a shadow or base for the letters.

# Questions

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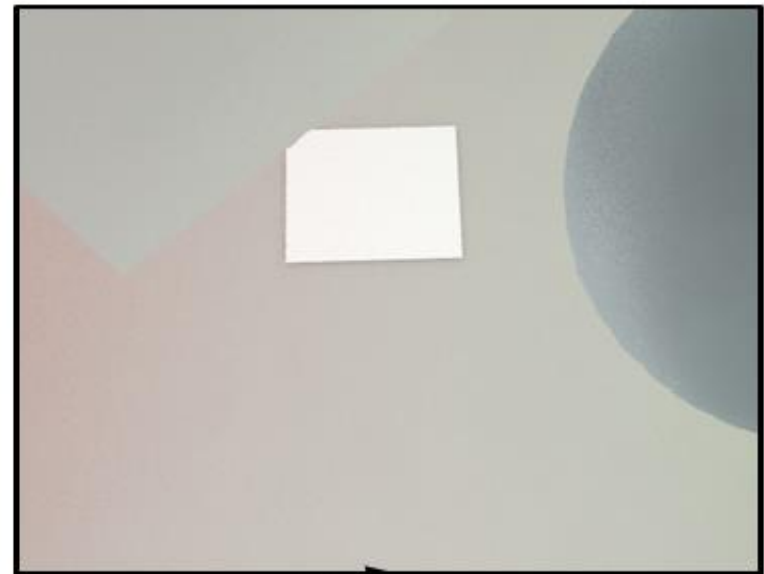
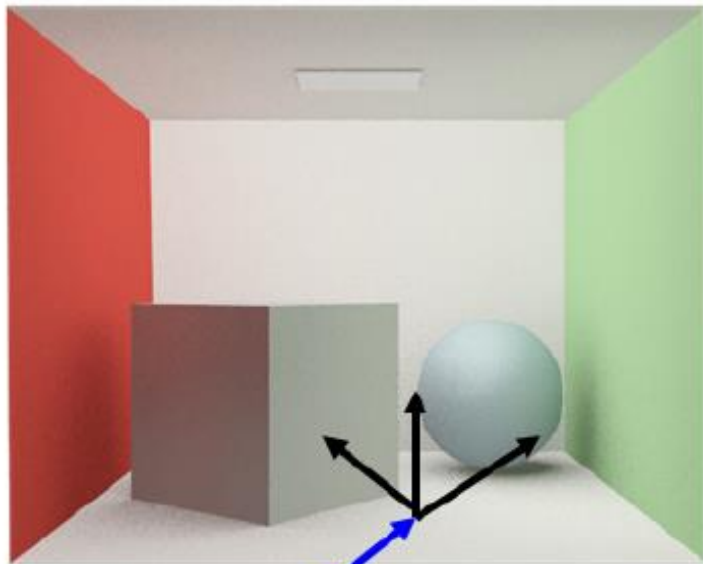
- **What is rule of thumb for choosing right amount of samples? Selecting few might result in bias and selecting too many would be meaningless since variance converges quickly. I wonder if there is some lower bound for sample numbers to get decent result.**

# Class Objectives (Ch. 15)

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- **Understand a basic structure of Monte Carlo ray tracing**
  - **Russian roulette for its termination**
  - **Path tracing**
- **Last time:**
  - **Monte Carlo integration: sampling approach for solving the rendering equation**
  - **Estimator and its variance**

# Rendering Equation

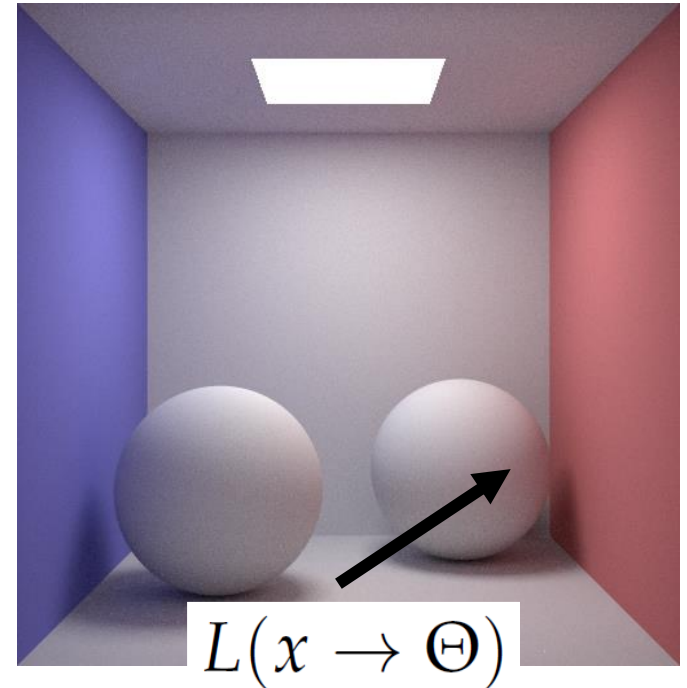


Incoming radiance on the hemisphere

$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x dw_{\Psi}$$

# Evaluation

- **To compute**  $L(x \rightarrow \Theta)$  :
  - **Check**  $L_e(x \rightarrow \Theta)$
  - **Evaluate**  $L_r(x \rightarrow \Theta)$



$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi}$$

# Evaluation

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- Use Monte Carlo
- Generate random directions on hemisphere  $\Psi$  using pdf  $p(\Psi)$

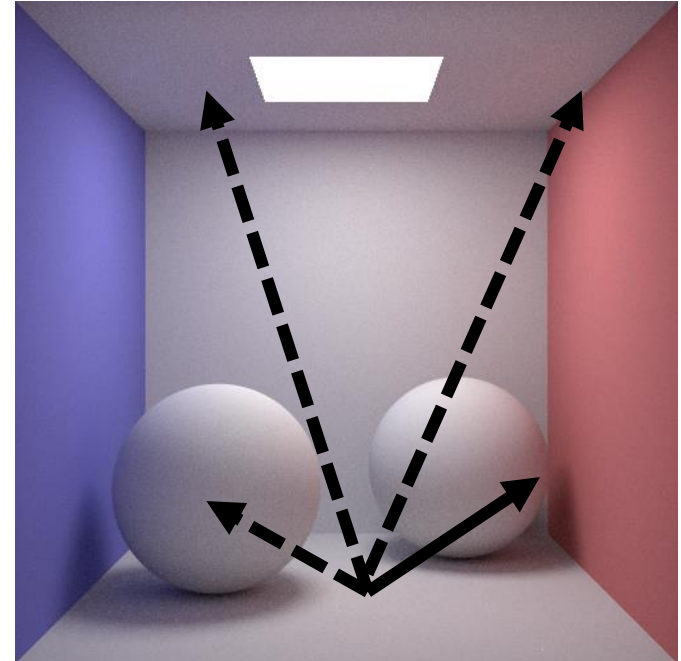
$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos \theta_x d\omega_{\Psi}$$

$$\hat{L}_r(x \rightarrow \Theta) = \frac{1}{N} \sum_{i=1}^N \frac{L(x \leftarrow \Psi_i) f_r(x, \Psi_i \rightarrow \Theta) \cos \theta_x}{p(\Psi_i)}$$

- How about  $L(x \leftarrow \Psi_i)$  ?

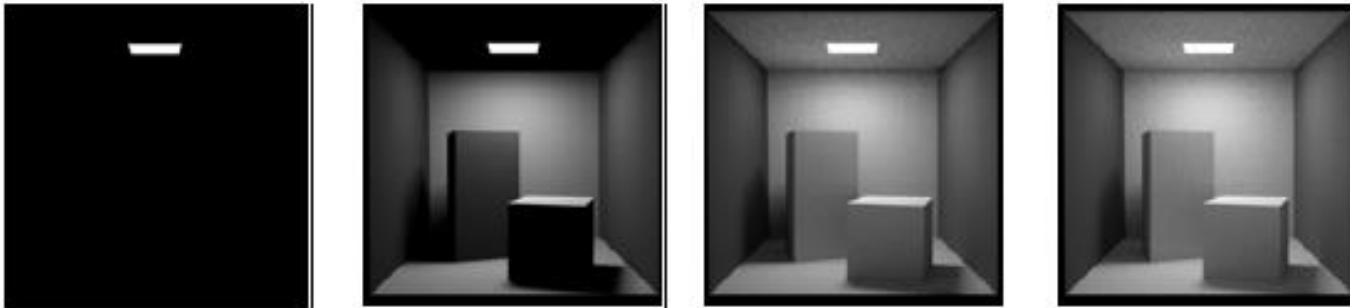
# Evaluation

- How about  $L(x \leftarrow \Psi_i)$  ?
- Perform ray casting backward
- Compute radiance from those visible points to  $x$ 
  - Assume reciprocity
- Recursively perform the process
  - Each additional bounce supports one more indirect illumination



# When to end recursion?

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From kavita's slides

- **Contributions of further light bounces become less significant**
  - Max recursion
  - Some threshold for radiance value
- **If we just ignore them, estimators will be biased**



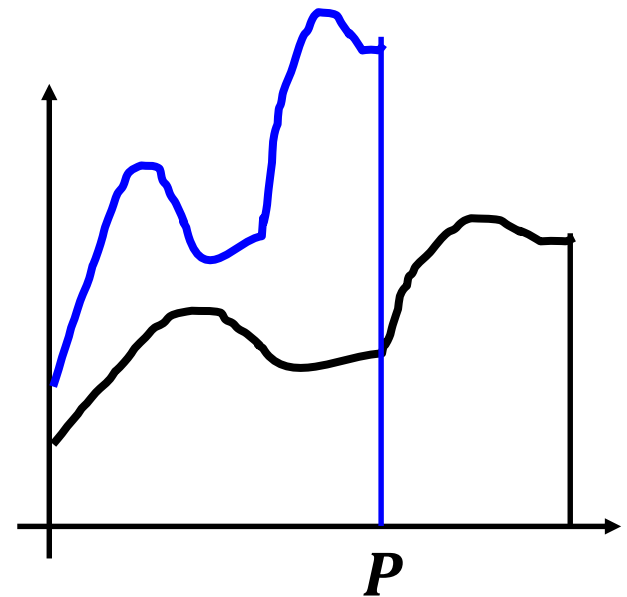
# Russian Roulette

- **Integral: Substitute  $y = Px$**

$$I = \int_0^1 f(x) dx = \int_0^P \frac{f(y/P)}{P} dy.$$

- **Estimator**

$$\hat{I}_{\text{roulette}} = \begin{cases} \frac{f(x_i)}{P} & \text{if } x_i \leq P, \\ 0 & \text{if } x_i > P. \end{cases}$$



- **Variance?**

# Russian Roulette

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- **Pick absorption probability,  $\alpha = 1-P$** 
  - **Recursion is terminated**
- **$1-\alpha$ , i.e.,  $P$ , is commonly to be equal to the reflectance of the material of the surface**
  - **Darker surface absorbs more paths**

# Algorithm so far

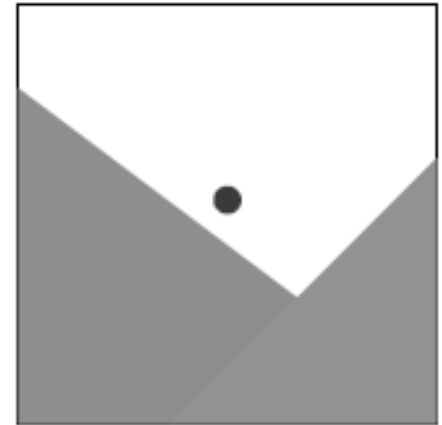
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- **Shoot primary rays through each pixel**
- **Shoot indirect rays, sampled over hemisphere**
- **Terminate recursion using Russian Roulette**

# Pixel Anti-Aliasing

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- **Compute radiance only at the center of pixel**
  - **Produce jaggies**
- **We want to evaluate using MC**
- **Simple box filter**
  - **The averaging method**

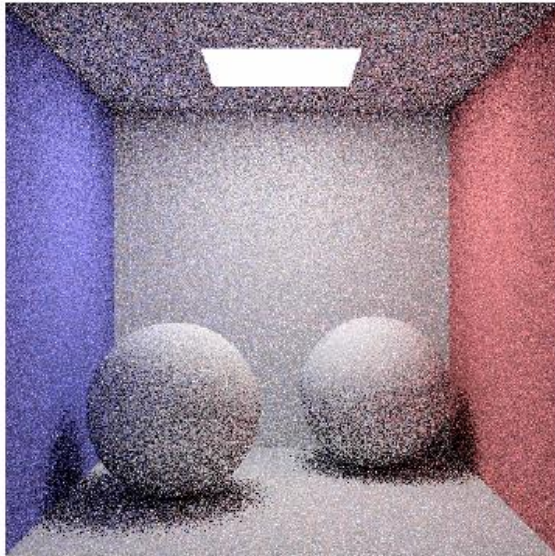


# Stochastic Ray Tracing

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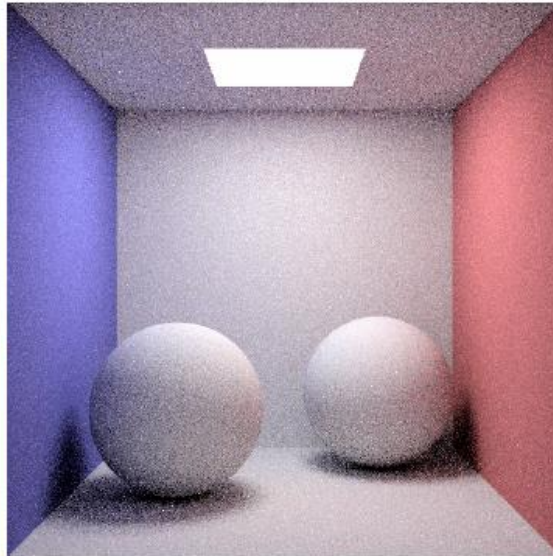
- **Parameters**
  - **Num. of starting ray per pixel**
  - **Num. of random rays for each surface point (branching factor)**
- **Path tracing**
  - **Branching factor = 1**

# Path Tracing

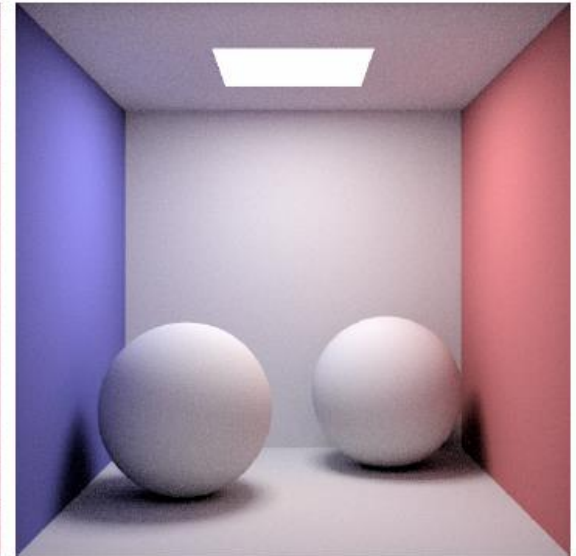


1 spp

(samples per pixel)



4 spp



16 spp

- **Pixel sampling + light source sampling folded into one method**

# Algorithm so far

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- **Shoot primary rays through each pixel**
- **Shoot indirect rays, sampled over hemisphere**
  - **Path tracing shoots only 1 indirect ray**
- **Terminate recursion using Russian Roulette**

# Performance

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- **Want better quality with smaller # of samples**
  - **Fewer samples/better performance**
  - **Quasi Monte Carlo: well-distributed samples**
  - **Adaptive sampling**



# Some Example

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**Uniform sampling  
(64 samples per pixel)**



**Adaptive sampling**

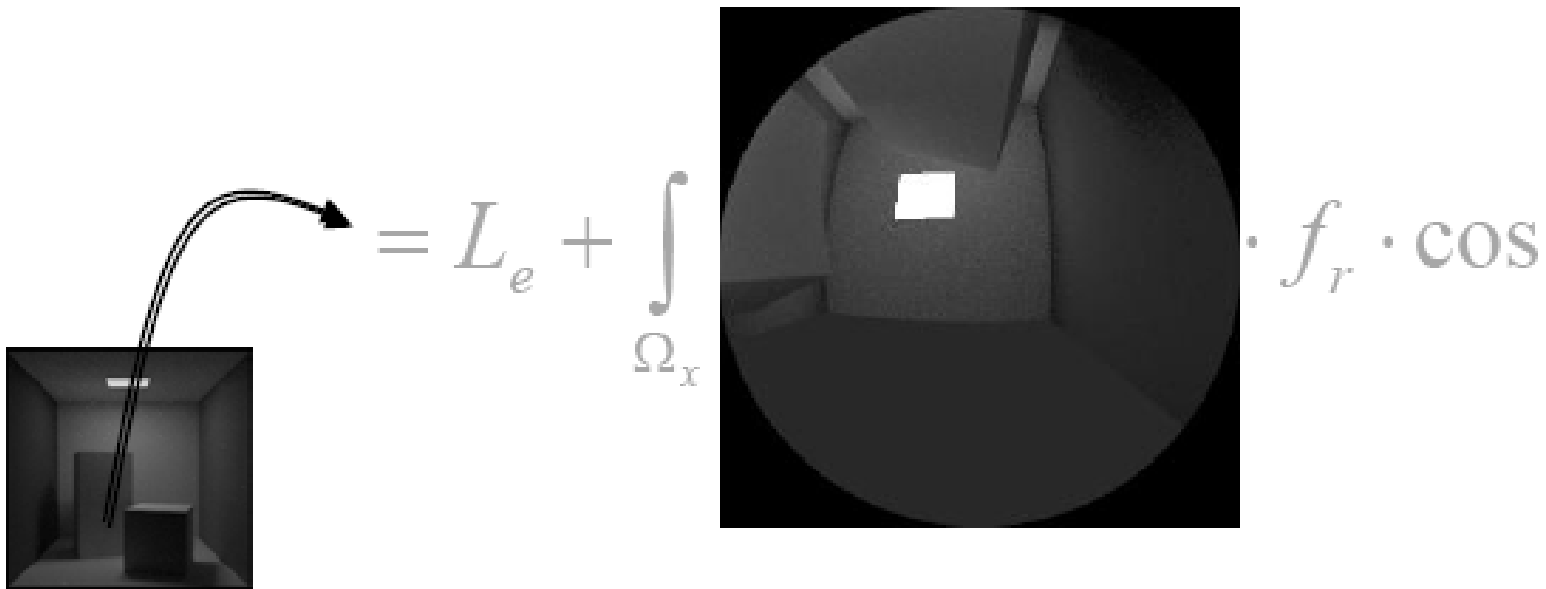


**Reference**

# Importance Sampling

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos(\Psi, n_x) \cdot d\omega_\Psi$$

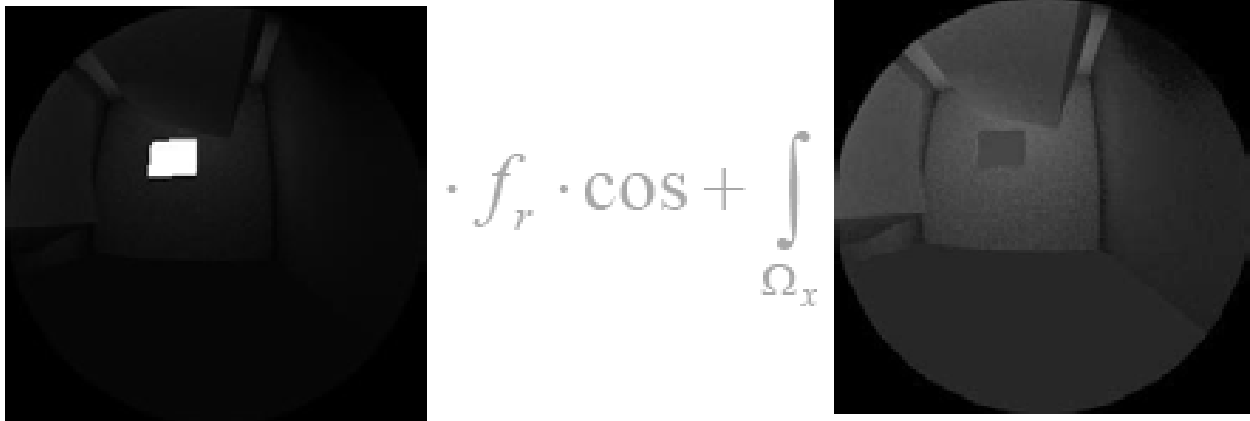
Radiance from light sources + radiance from other surfaces



# Importance Sampling

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$$L(x \rightarrow \Theta) = L_e + L_{direct} + L_{indirect}$$

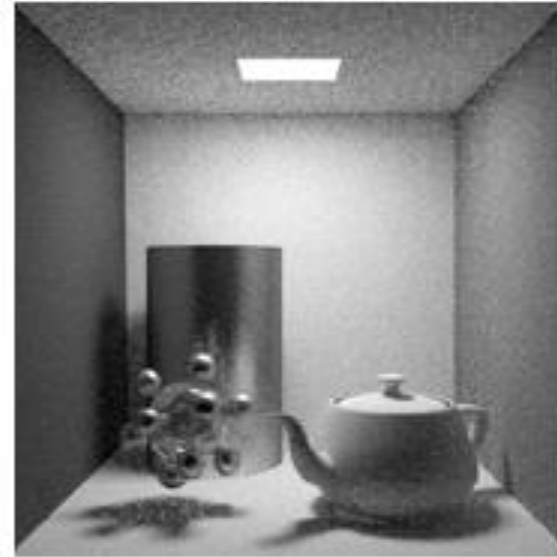
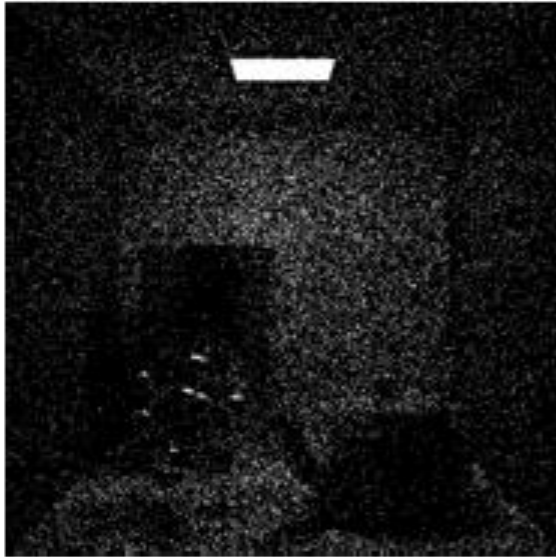
$$= L_e + \int_{\Omega_x} \text{img}_1 \cdot f_r \cdot \cos + \int_{\Omega_x} \text{img}_2 \cdot f_r \cdot \cos$$


- So ... sample direct and indirect with separate MC integration



# Comparison

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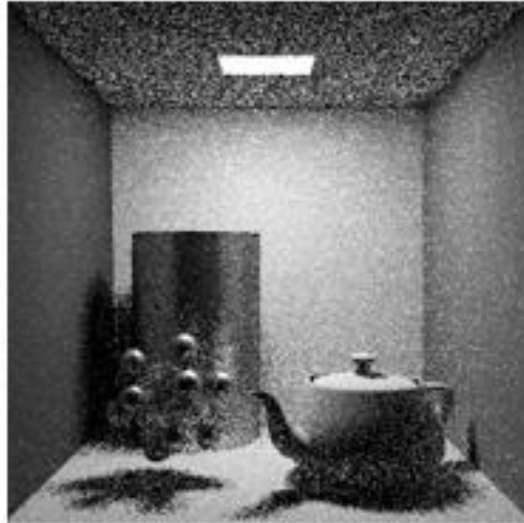
From kavita's slides

- **With and without considering direct illumination**
  - **16 samples / pixel**

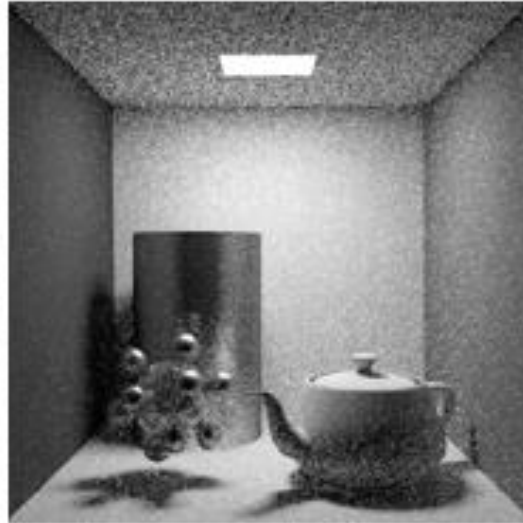
# Rays per pixel

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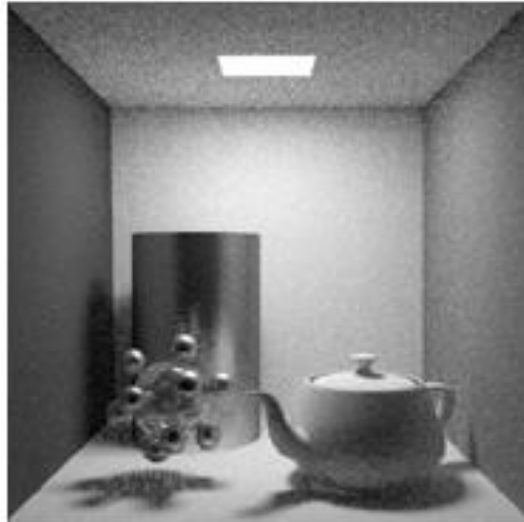
1 sample/  
pixel



4 samples/  
pixel



16 samples/  
pixel



256 samples/  
pixel



# Class Objectives were:

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- **Understand a basic structure of Monte Carlo ray tracing**
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# Next Time...

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- **Acceleration techniques for global illumination methods**



# Homework

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- **Go over the next lecture slides before the class**
- **Watch 2 SIG/CVPR/ISMAR videos and submit your summaries every Mon. class**
  - **Just one paragraph for each summary**
  - **Any top-tier conf (e.g., ICRA) is okay**

## Example:

**Title: XXX XXXX XXXX**

**Abstract: this video is about accelerating the performance of ray tracing. To achieve its goal, they design a new technique for reordering rays, since by doing so, they can improve the ray coherence and thus improve the overall performance.**

# Any Questions?

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- **Submit four times in Sep./Oct.**
- **Come up with one question on what we have discussed in the class and submit at the end of the class**
  - **1 for typical questions**
  - **2 for questions that have some thoughts or surprise me**