CS380: Computer Graphics
Triangle Rasterization

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(윤성의)

Course URL:
http://sglab.kaist.ac.kr/~sungeui/CG/

Class Objectives

- Understand triangle rasterization using edge-equations
- Understand mechanics for parameter interpolations
- Realize benefits of incremental algorithms

Topics for Today

- Quick review of coordinate systems
- Motivation
  - What is rasterization?
  - Why triangles?
- Rasterization
  - Scan-line
  - Edge equations
- Interpolation
- Beyond triangles

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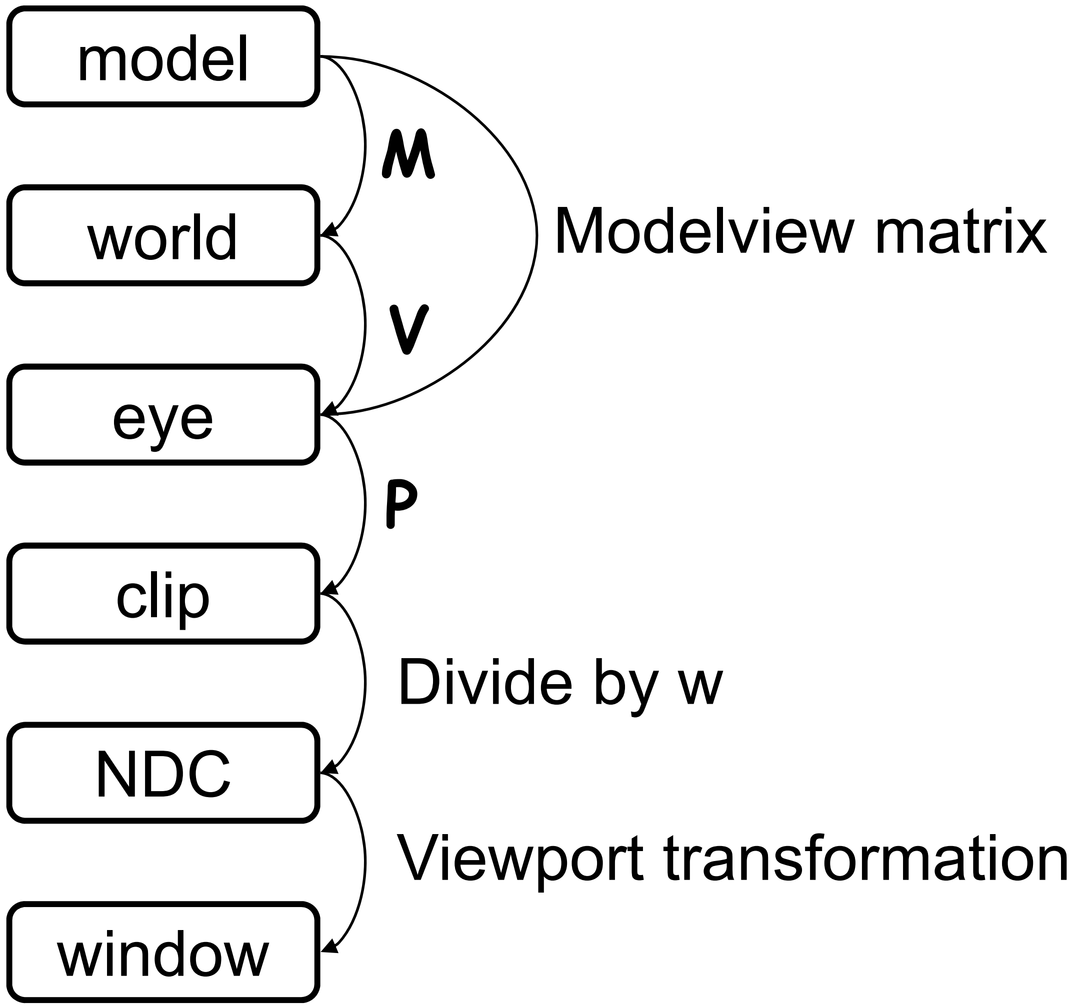
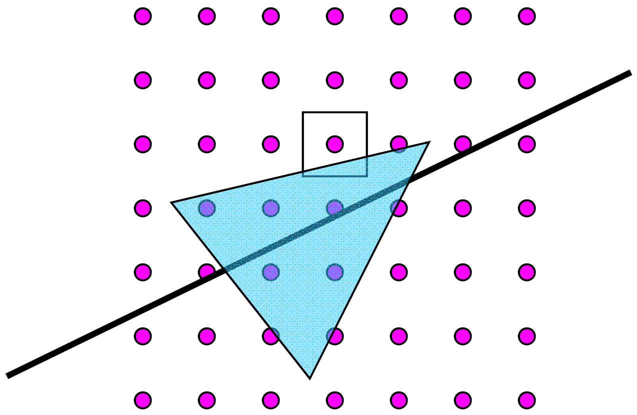
Coordinate Systems



Primitive Rasterization

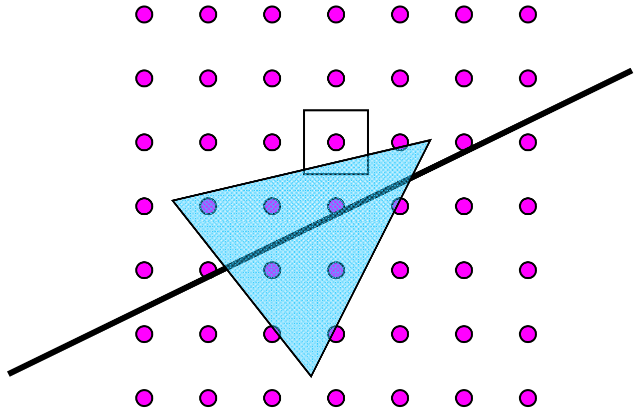
- Rasterization converts vertex representation to pixel representation



- Coverage determination
  - Computes which pixels (samples) belong to a primitive
- Parameter interpolation
  - Computes parameters at covered pixels from parameters associated with primitive vertices

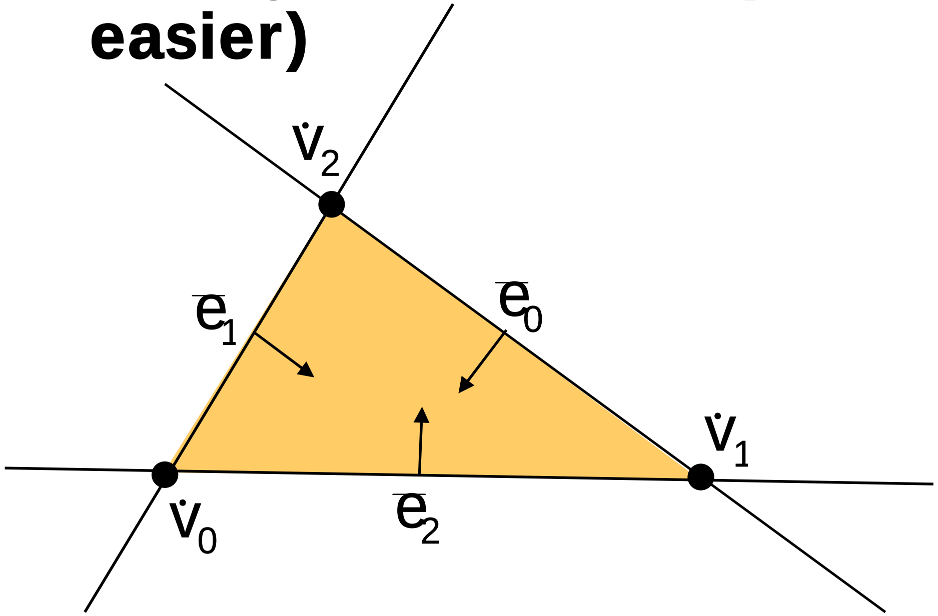
Coverage Determination

- **Coverage is a 2D sampling problem**
- **Possible coverage criteria:**
  - **Distance of the primitive to sample point (often used with lines)**
  - **Percent coverage of a pixel (used to be popular)**
  - **Sample is inside the primitive (assuming it is closed)**



Why Triangles?

- Triangles are simple
  - Simple representation for a surface element (3 points or 3 edge equations)
  - Triangles are linear (makes computations easier)

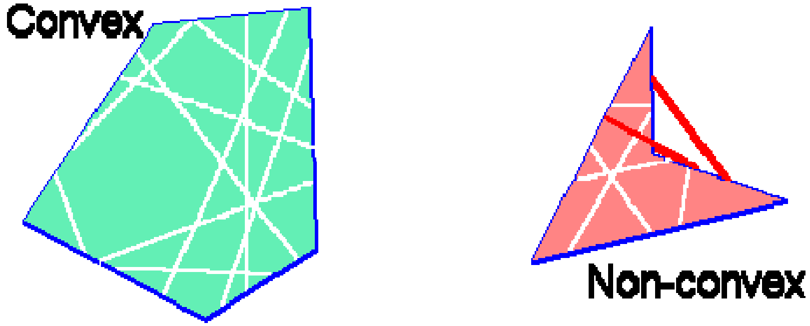


$$T = (\dot{v}_0, \dot{v}_1, \dot{v}_2)$$

$$T = (\bar{e}_0, \bar{e}_1, \bar{e}_2)$$

Why Triangles?

- Triangles are convex
- What does it mean to be a convex?



An object is convex if and only if any line segment connecting two points on ts boundary is contained entirely within the object or one of its boundaries

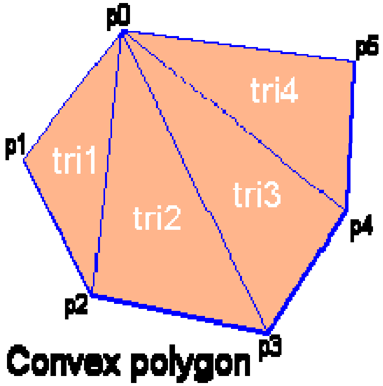
Why Triangles?

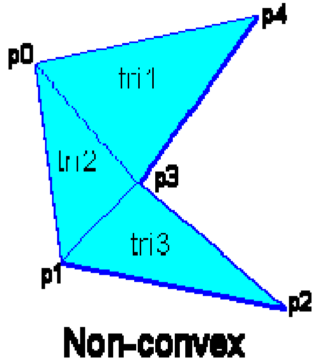
- Triangles are convex

- Why is convexity important?
  - Regardless of a triangle's orientation on the screen a given scan line will contain only a single segment or *span* of that triangle
  - Simplify rasterization processes

Why Triangles?

- Arbitrary polygons can be decomposed into triangles

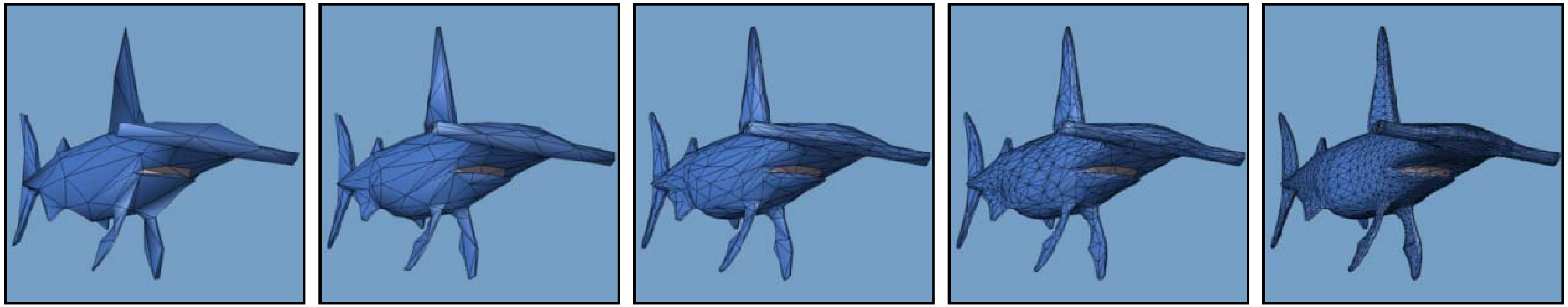




- Decomposing a convex n-sided polygon is trivial
  - Suppose the polygon has ordered vertices $\{v_0, v_1, \dots v_n\}$
  - It can be decomposed into triangles $\{(v_0,v_1,v_2), \{v_0,v_2,v_3), (v_0,v_i,v_{i+1}), \dots (v_0,v_{n-1},v_n)\}$
- Decomposing a non-convex polygon is non-trivial
  - Sometimes have to introduce new vertices

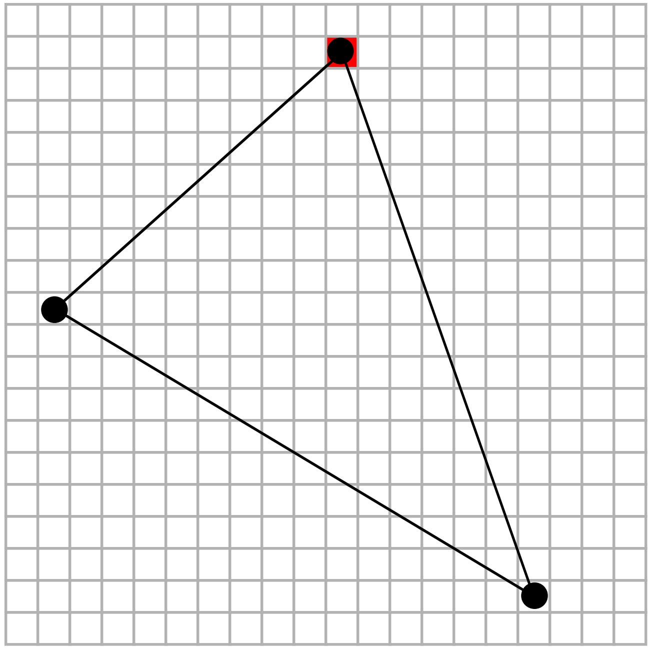
Why Triangles?

- **Triangles can approximate any 2-dimensional shape (or 3D surface)**
  - **Polygons are a locally linear (planar) approximation**
- **Improve the quality of fit by increasing the number edges or faces**



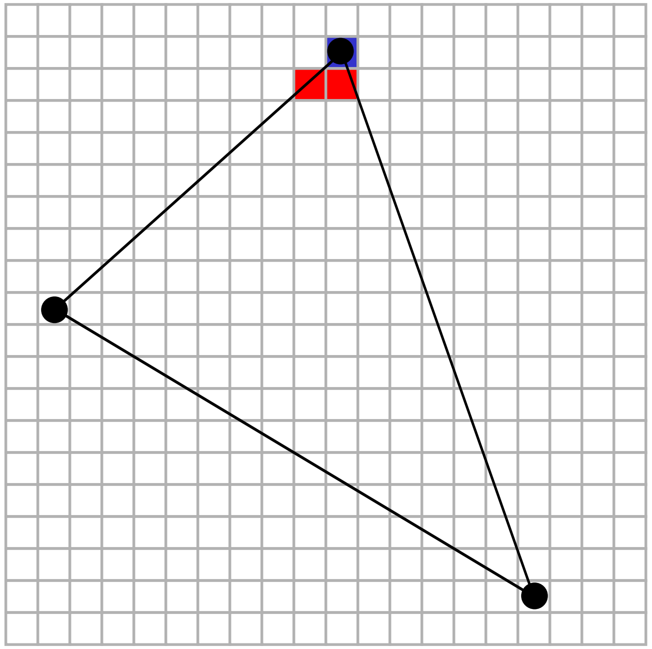
Scanline Triangle Rasterizer

- Walk along edges and process one scanline at a time; also called edge walk method
- Rasterize spans between edges



Scanline Triangle Rasterizer

- **Walk along edges and process one scanline at a time**
- **Rasterize spans between edges**



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Scanline Triangle Rasterizer

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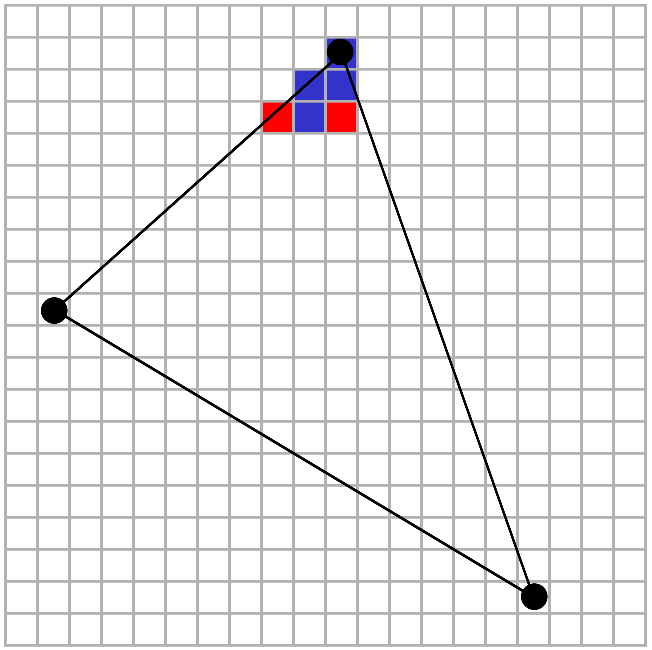

Scanline Triangle Rasterizer

- **Walk along edges and process one scanline at a time**
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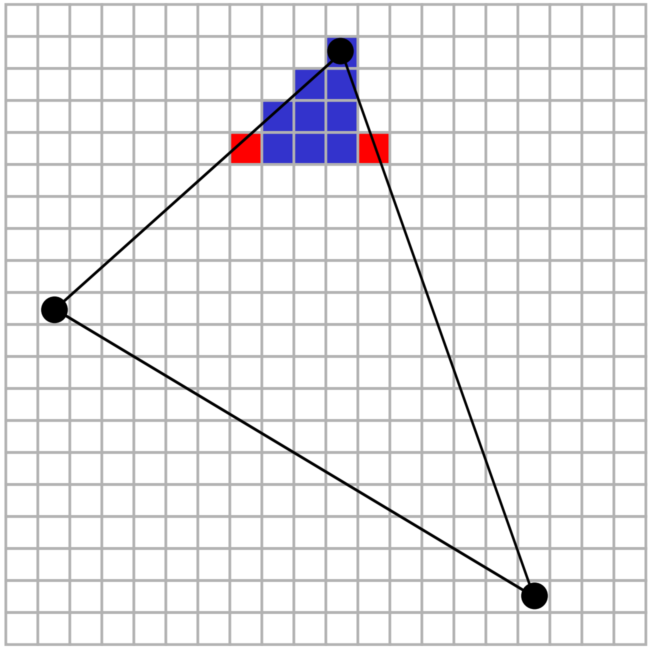

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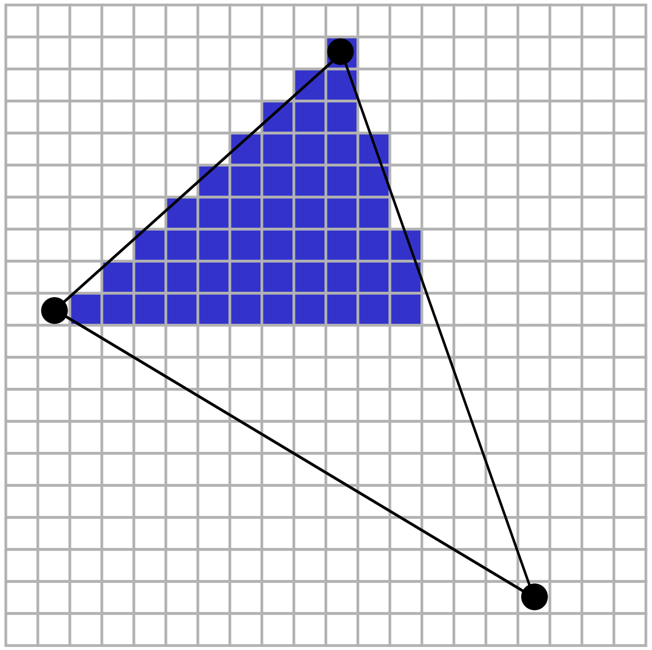

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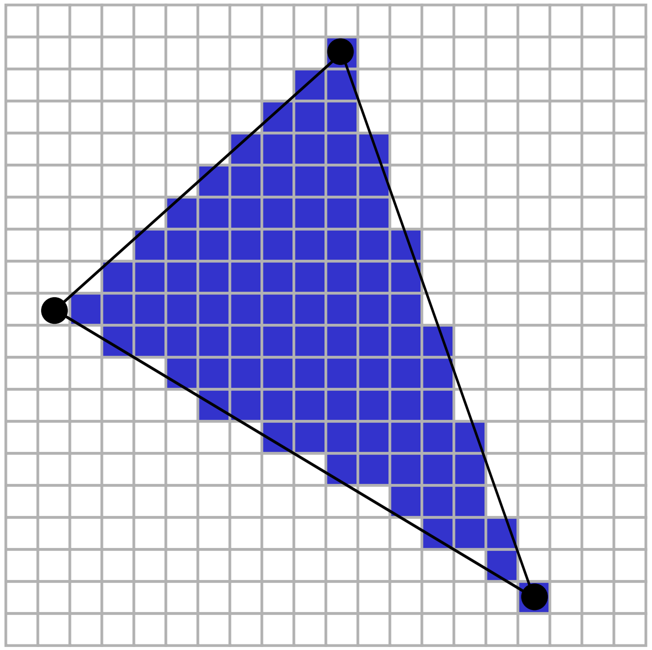
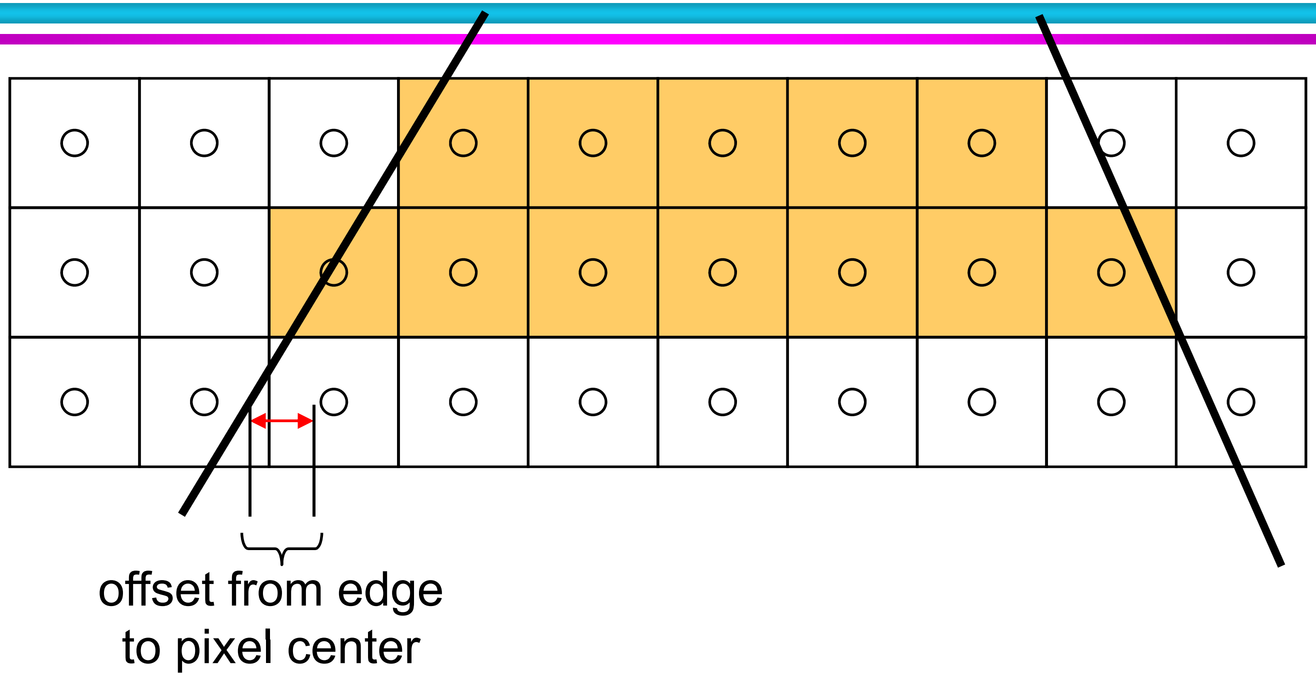


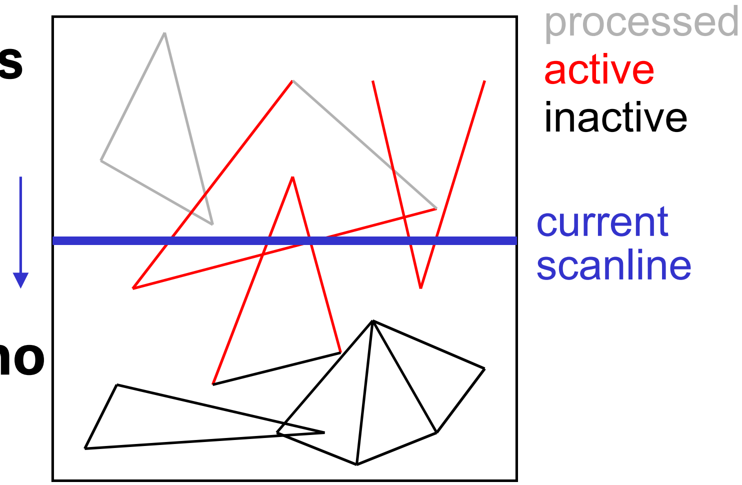
Fractional Offsets



- **Straightforward to interpolate values (e.g. colors) along the edges, but must be careful when offsetting from the edge to the pixel's center**

Scanline Rasterizing Scenes

- **Sort all edges by start scanline into the Inactive Edge Table (IET)**
- **Move edges intersected by current scanline from IET to Active Edge Table (AET)**
- **Compute spans between active edges**
- **Sort spans by starting x**
- **Rasterize visible span segments**
- **Remove edges from AET when they no longer intersect the current scanline**

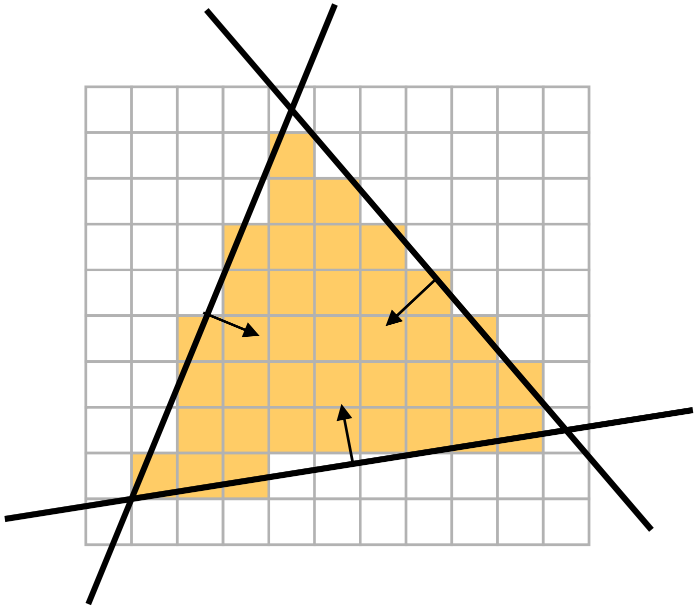


Scanline Rasterization

- **Advantages:**
  - **Can be made quite fast**
  - **Low memory usage for small scenes**
  - **Do not need full 2D z-buffer (can use 1D z-buffer on the scanline)**
- **Disadvantages:**
  - **Does not scale well to large scenes**
  - **Have to worry about fractional offsets**
  - **Lots of special cases**

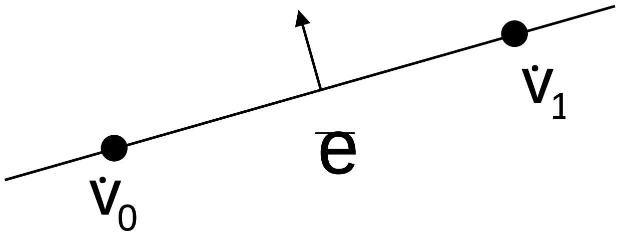
Rasterizing with Edge Equations

- Compute edge equations from vertices
- Compute interpolation equations from vertex parameters
- Traverse pixels evaluating the edge equations
- Draw pixels for which all edge equations are positive
- Interpolate parameters at pixels



Edge Equation Coefficients

- **The cross product between 2 homogeneous points generates the line between them**



$$\overline{e} = \dot{v}_0 \times \dot{v}_1$$

$$= [x_0 \quad y_0 \quad 1]^t \times [x_1 \quad y_1 \quad 1]^t$$

$$= [(y_0 - y_1) \quad (x_1 - x_0) \quad (x_0 y_1 - x_1 y_0)]$$

$$\qquad A \qquad\qquad B \qquad\qquad C$$

$$E(x,y) = Ax + By + C$$

- **A pixel at (x,y) is "inside" an edge if $E(x,y) > 0$**

Numerical Precision

- **Subtraction of two nearly equal floating point numbers results in catastophic cancellation which leaves only a few significant bits**

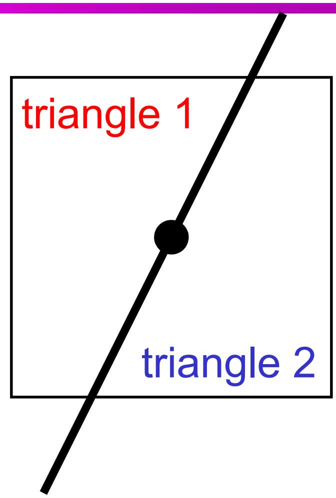
$$1234 \times 10^3 - 1233 \times 10^3 = 1000 \times 10^0$$

- **When $x_0 y_1 \approx x_1 y_0$ computing $C = x_0 y_1 - x_1 y_0$ can result in loss of precision**
- **Reformulate C coeffcient:**

$$C = -\frac{A(x_0 + x_1) + B(y_0 + y_1)}{2}$$

Shared Edges

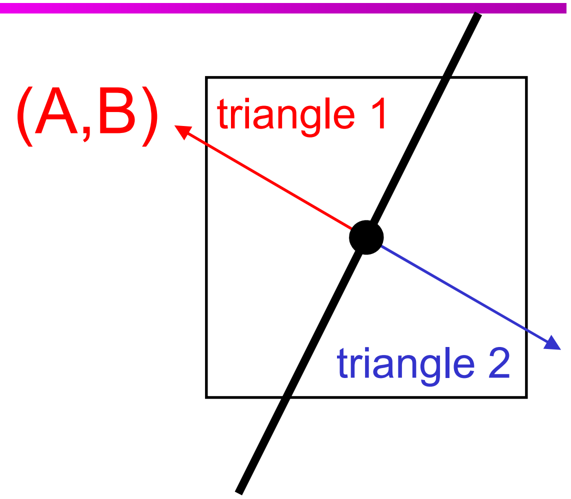
- **Suppose two triangles share an edge. Which covers the pixel when the edge passes through the sample (E(x,y)=0)?**
- **Both**
  - **Pixel color becomes dependent on order of triangle rendering**
  - **Creates problems when rendering transparent objects - "double hitting"**
- **Neither**
  - **Missing pixels create holes in otherwise solid surface**
- **We need a consistent tie-breaker!**



Shared Edges

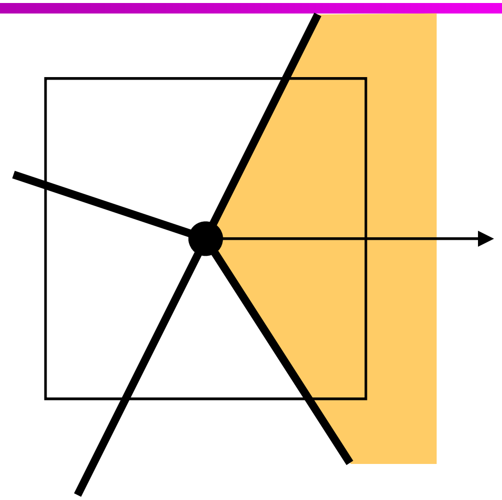
- **A common tie-breaker:**

$$\text{bool } t = \begin{cases} A > 0 & \text{if } A \neq 0 \\ B > 0 & \text{otherwise} \end{cases}$$

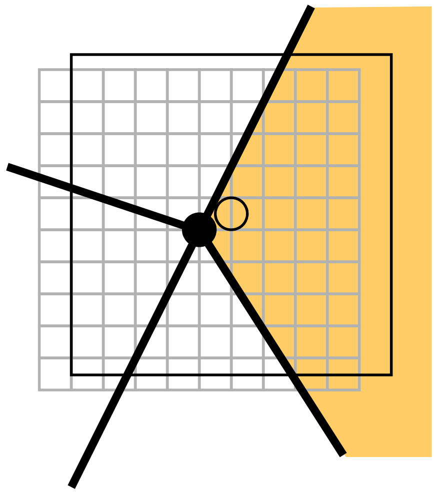


- **Coverage determination becomes**
  if(E(x,y) >0 || (E(x,y)==0 && t))
  pixel is covered

Shared Vertices



- Use "inclusion direction" as a tie breaker
- Any direction can be used



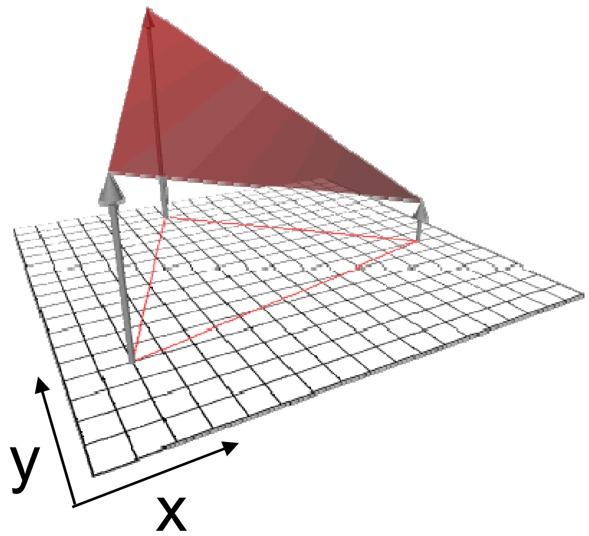
- Snap vertices to subpixel grid and displace so that no vertex can be at the pixel center

Other benefits of snapping to subpixel grid

- Simplicity
  - can use fixed-point arithmetic can be used (integer operations)
- Robustness
  - With sufficient bits, edge equations and areas can be computed exactly
- Quality
  - Smoother animation than if we snapped to the pixel grid

Interpolating Parameters

- **Specify a parameter, say redness (r) at each vertex of the triangle**
  - **Linear interpolation creates a planar function**



$$r(x,y) = Ax + By + C$$

Solving for Linear Interpolation Equations

- **Given the redness of the three vertices, we can set up the following linear system:**

$$\begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} = \begin{bmatrix} A_r & B_r & C_r \end{bmatrix} \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$$

with the solution:

$$\begin{bmatrix} A_r & B_r & C_r \end{bmatrix} = \begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} \frac{\begin{bmatrix} (y_1 - y_2) & (x_2 - x_1) & (x_1 y_2 - x_2 y_1) \\ (y_0 - y_2) & (x_2 - x_0) & (x_0 y_2 - x_2 y_0) \\ (y_0 - y_1) & (x_1 - x_0) & (x_0 y_1 - x_1 y_0) \end{bmatrix}}{\det \begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}}$$

Triangle Area

$$\text{Area} = \tfrac{1}{2}\det\begin{bmatrix} x_0 & x_1 & x_2 \\ y_0 & y_1 & y_2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \tfrac{1}{2}((x_1 y_2 - x_2 y_1) - (x_0 y_2 - x_2 y_0) + (x_0 y_1 - x_1 y_0))$$

$$= \tfrac{1}{2}(C_0 + C_1 + C_2)$$

- **Area = 0 means that the triangle is not visible**
- **Area < 0 means the triangle is back facing:**
  - **Reject triangle if performing back-face culling**
  - **Otherwise, flip edge equations by multiplying by -1**

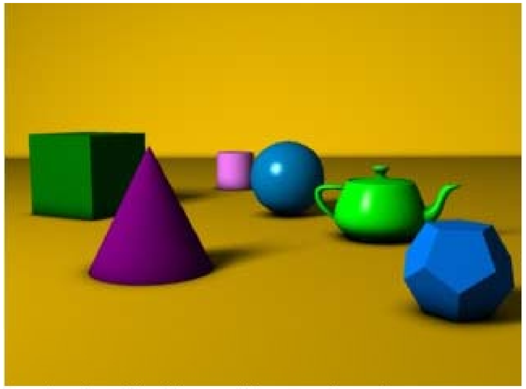
Interpolation Equation

- **The parameter plane equation is just a linear combination of the edge equations**

$$\begin{bmatrix} A_r & B_r & C_r \end{bmatrix} = \frac{1}{2 \cdot \text{area}} \begin{bmatrix} r_0 & r_1 & r_2 \end{bmatrix} \begin{bmatrix} \overline{e}_0 \\ \overline{e}_1 \\ \overline{e}_2 \end{bmatrix}$$

Z-Buffering

- When rendering multiple triangles we need to determine which triangles are visible
- Use z-buffer to resolve visibility
  - Stores the depth at each pixel
- Initialize z-buffer to 1
  - Post-perspective z values lie between 0 and 1
- Linearly interpolate depth (z_{tri}) across triangles
- If $z_{tri}(x,y)$ < zBuffer[x][y]
  write to pixel at (x,y)
  zBuffer[x][y] = $z_{tri}(x,y)$

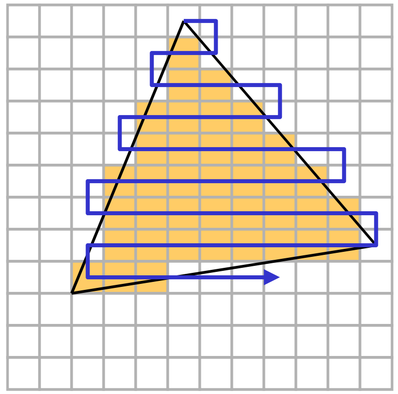

A simple three dimensional scene

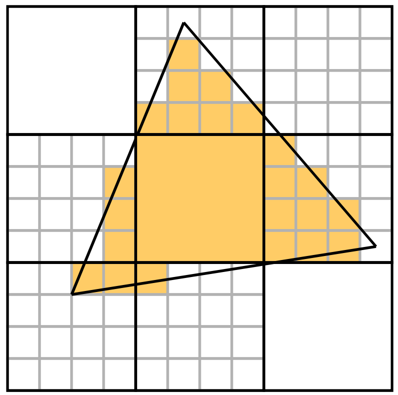

Z-buffer representation

image from wikipedia.com

Traversing Pixels

- Free to traverse pixels
  - Edge and interpolation equations can be computed at any point
- Try to minimize work
  - Restrict traversal to primitive bounding box
  - Hierarchical traversal
    - Knock out tiles of pixels (say 4x4) at a time
    - Test corners of tiles against equations
    - Test individual pixels of tiles not entirely inside or outside





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Incremental Algorithms

- Some computation can be saved by updating the edge and interpolation equations incrementally:

$$
\begin{aligned}
E(x,y) &= Ax + By + C \\
E(x+\Delta,y) &= A(x+\Delta) + By + C \\
&= E(x,y) + A\cdot\Delta \\
E(x,y+\Delta) &= Ax + B(y+\Delta) + C \\
&= E(x,y) + B\cdot\Delta
\end{aligned}
$$

- Equations can be updated with a single addition!

Triangle Setup

- Compute edge equations
  - 3 cross products
- Compute triangle area
  - A few additions
- Cull zero area and back-facing triangles and/or flip edge equations
- Compute interpolation equations
  - Matrix/vector product per parameter

Massive Models

100,000,000 primitives

1,000,000 pixels

100 visible primitives/pixel

- **Cost to render a single triangle**
  - **Specify 3 vertices**
  - **Compute 3 edge equations**
  - **Evaluate equations one**

St. Mathew models consisting of about 400M triangles (Michelangelo Project)

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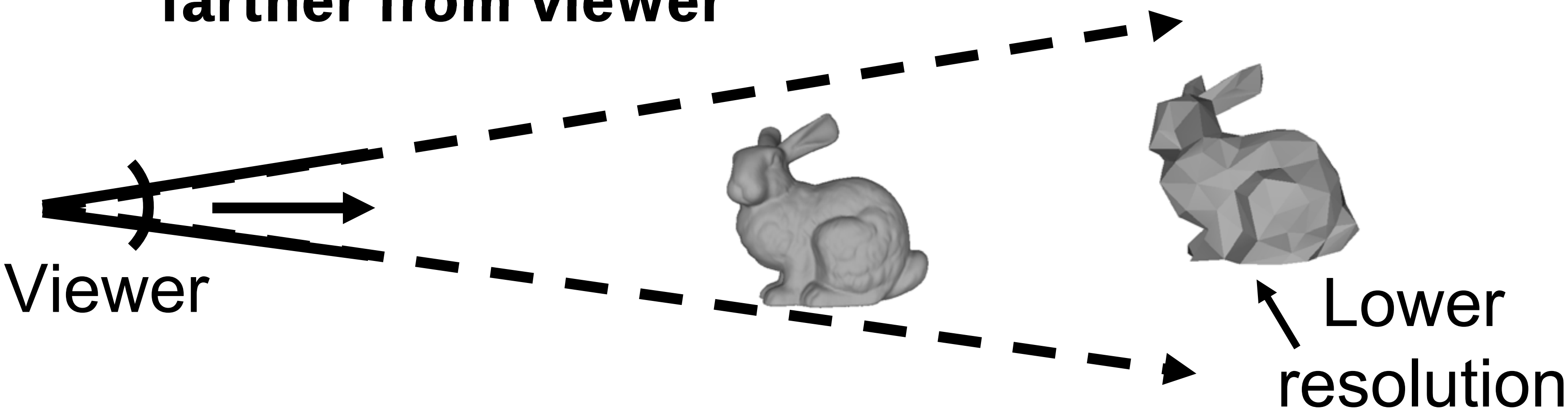
Multi-Resolution or Levels-of-Detail (LOD) Techniques

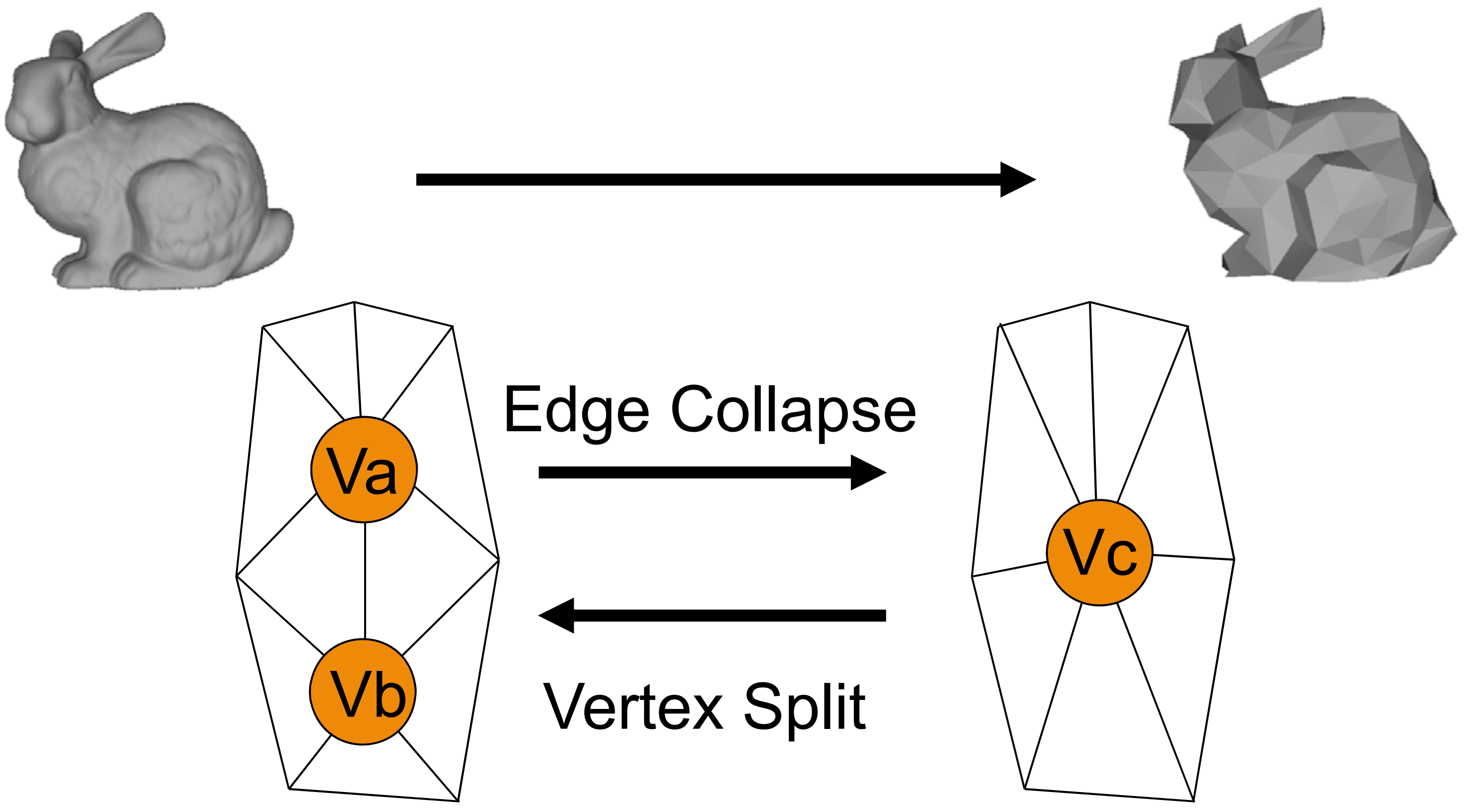
- **Basic idea**
  - **Render with fewer triangles when model is farther from viewer**



- **Methods**
  - **Polygonal simplification**

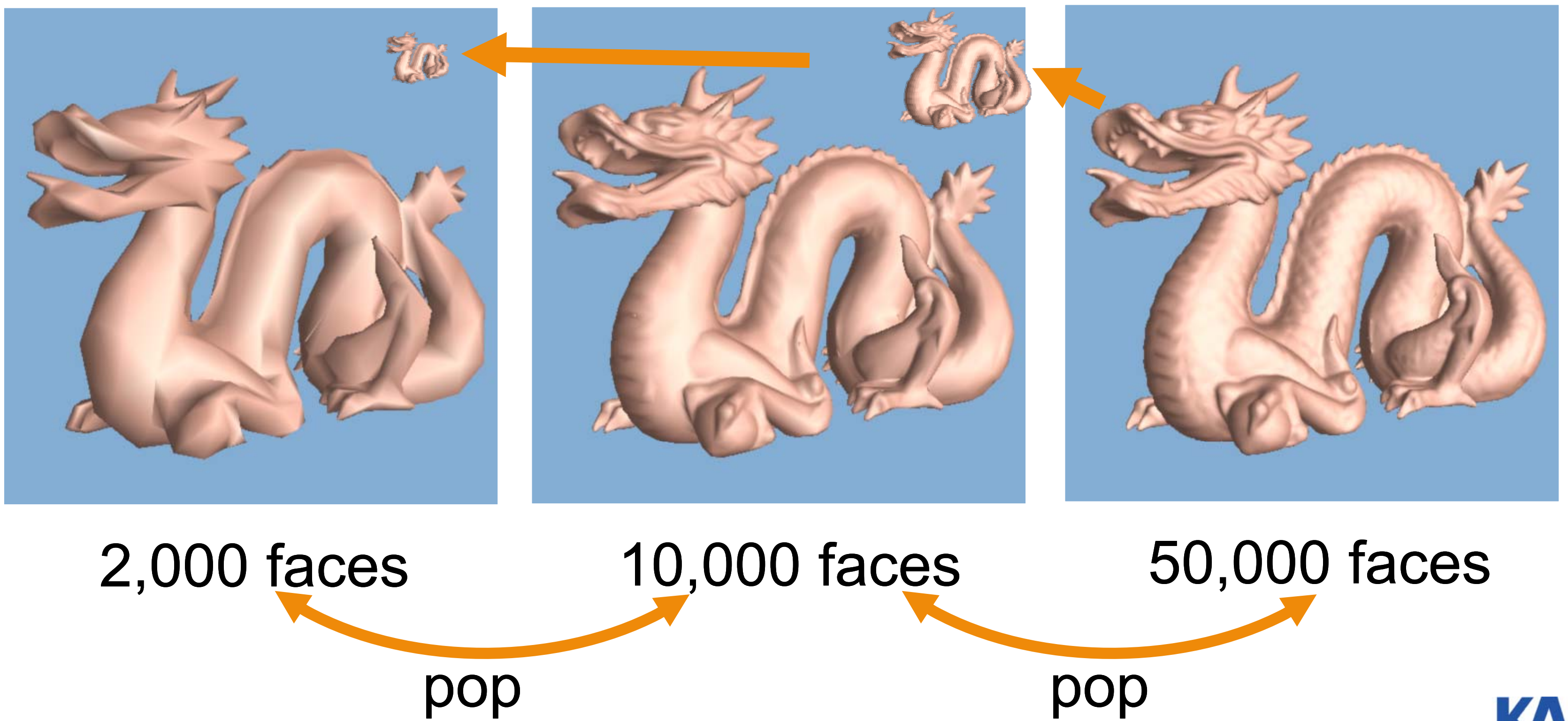
Polygonal Simplification

- **Method for reducing the polygon count of mesh**



Static LODs

- Pre-compute discrete simplified meshes
  - Switch between them at runtime
  - Has very low LOD selection overhead



Excerpted from Hoppe's slides

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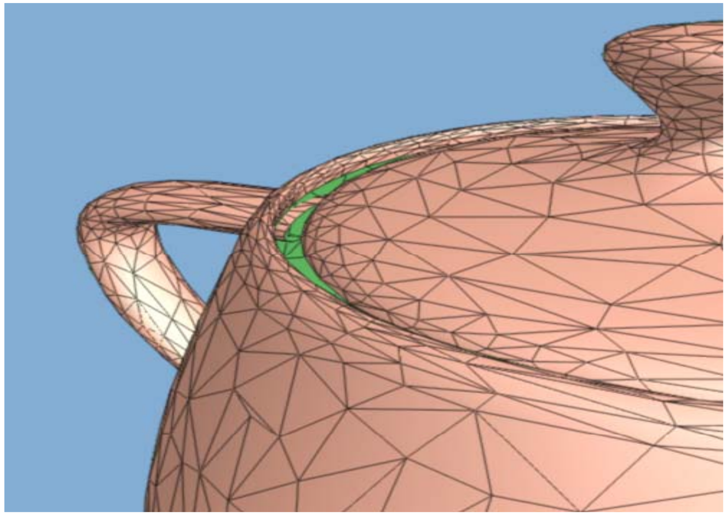
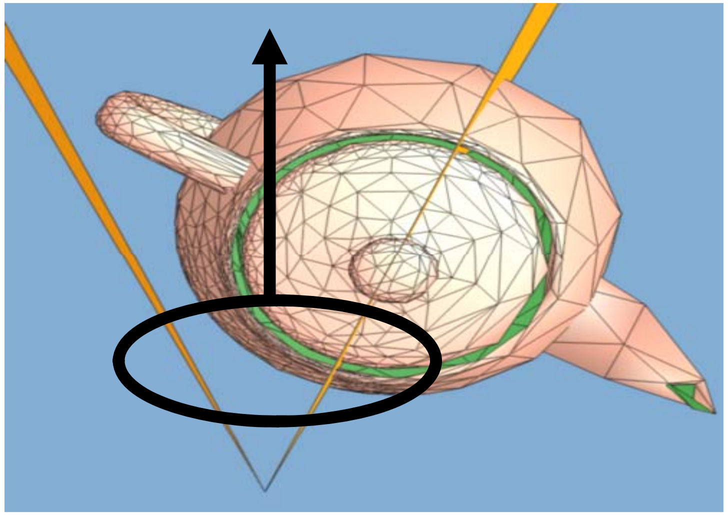
Dynamic Simplification

- **Provides smooth and varying LODs over the mesh** **[Hoppe 97]**

1st person's view | 3rd person's view





Play video

View-Dependent Rendering [Yoon et al., SIG 05]

30 Pixels of error

Pentium 4

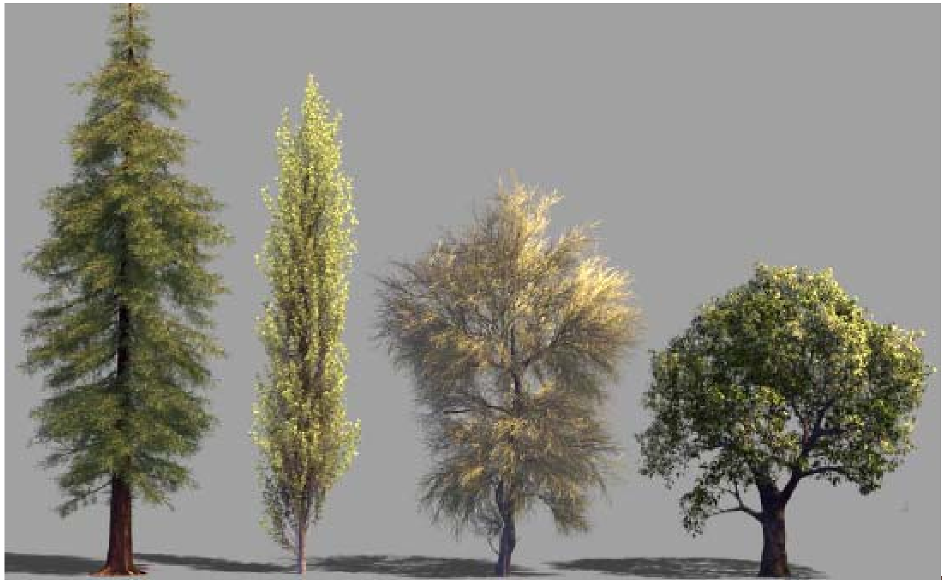
GeForce Go 6800 Ultra

1GB RAM

What if there are so many objects?

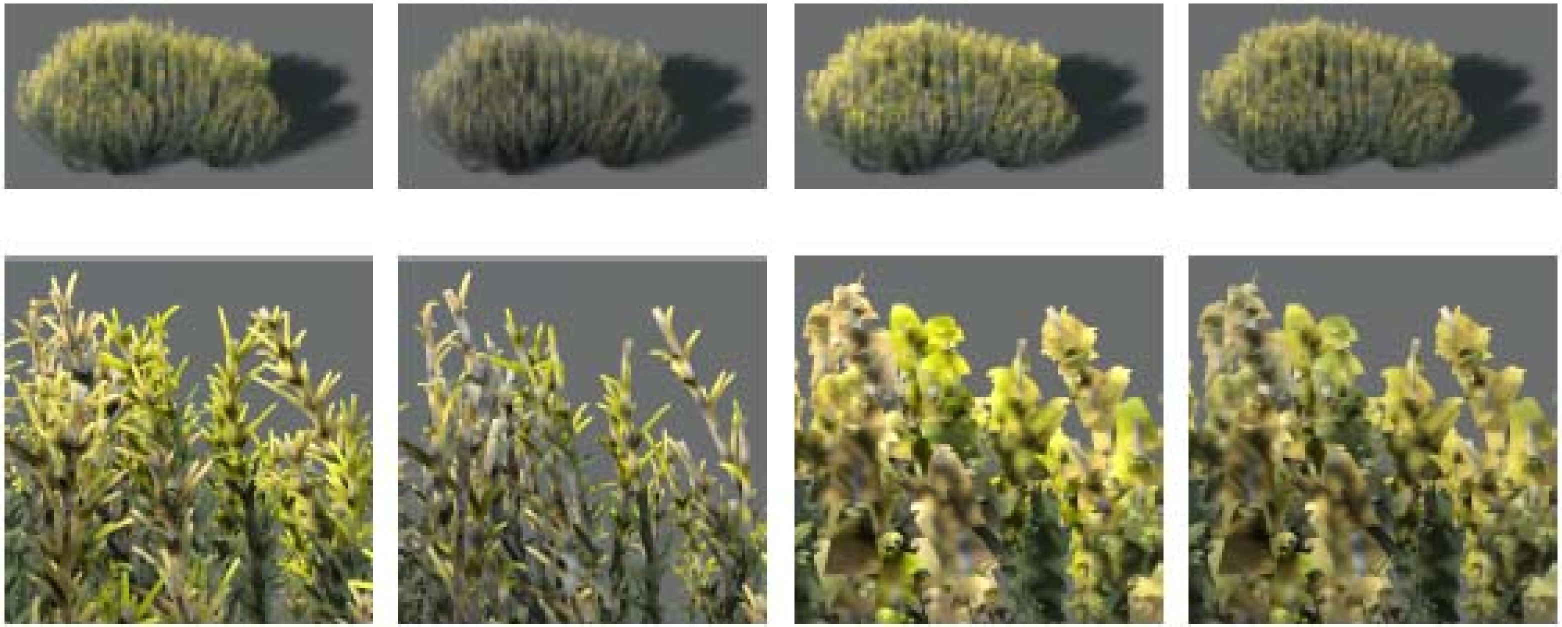
From "cars", a Pixar movie

What if there are so many objects?



From a Pixar movie

Stochastic Simplification of Aggregate Detail
Cook et al., ACM SIGGRAPH 2007

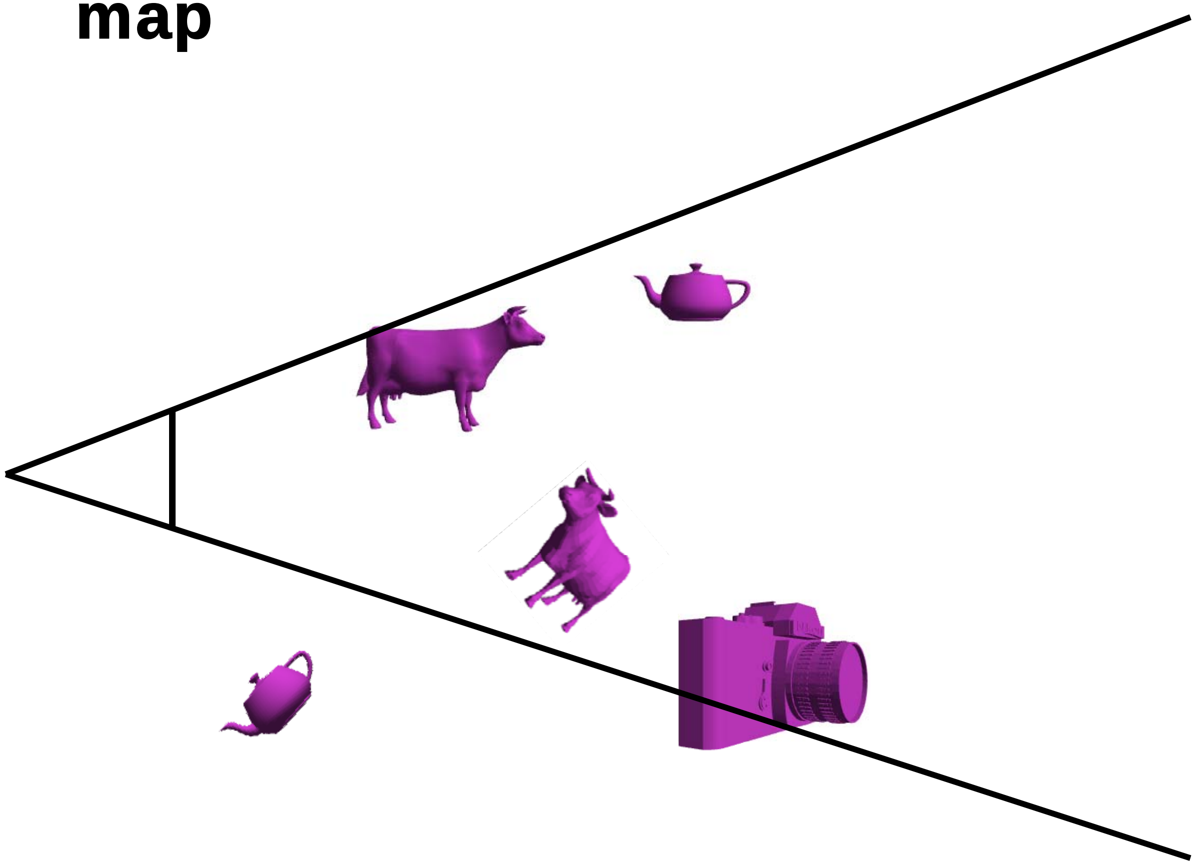


(a) (b) (c) (d)

Figure 2: *Distant views of the plant from Figure 1 with close-ups below: (a) unsimplified, (b) with 90% of its leaves excluded, (c) with area correction, (d) with area and contrast correction.*

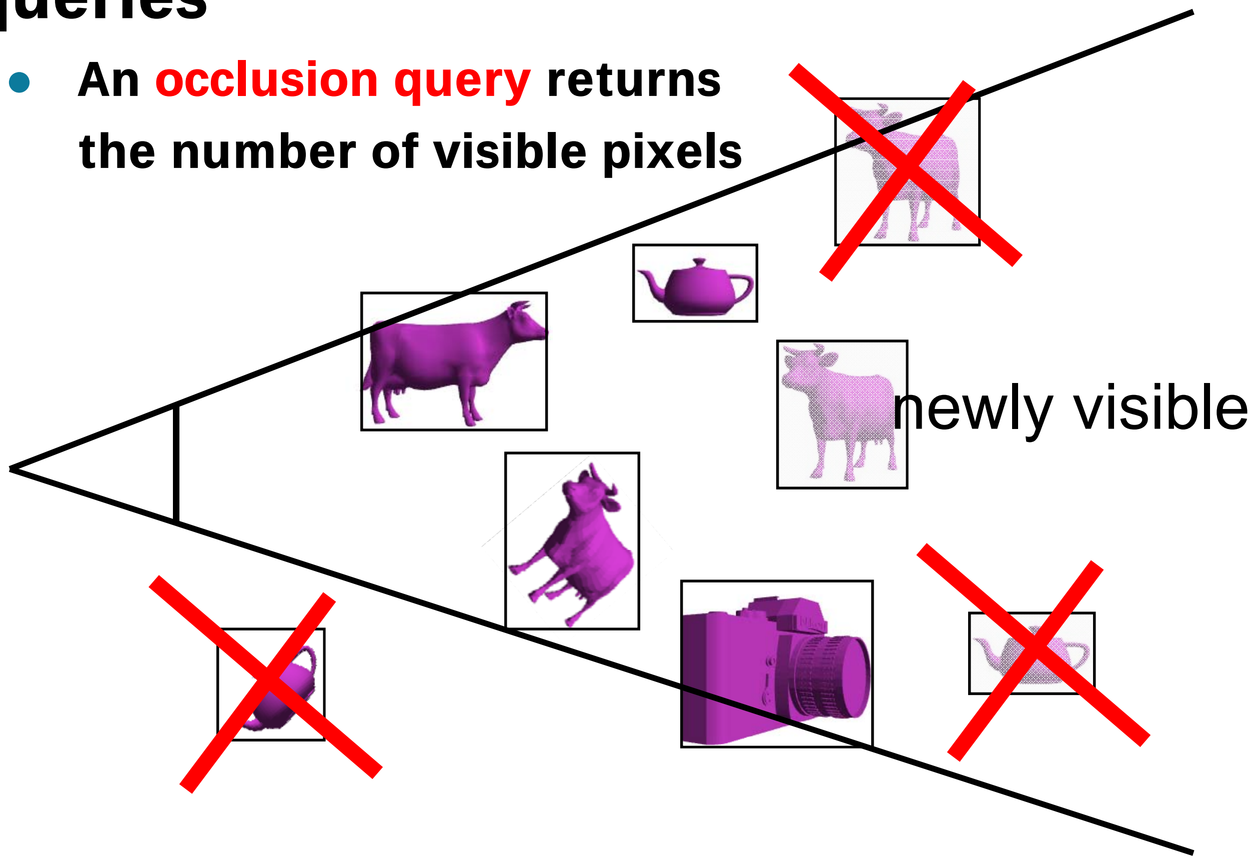
Occlusion Culling with Occlusion Queries

- Render objects visible in previous frame
  - Known as occlusion representation or occlusion map



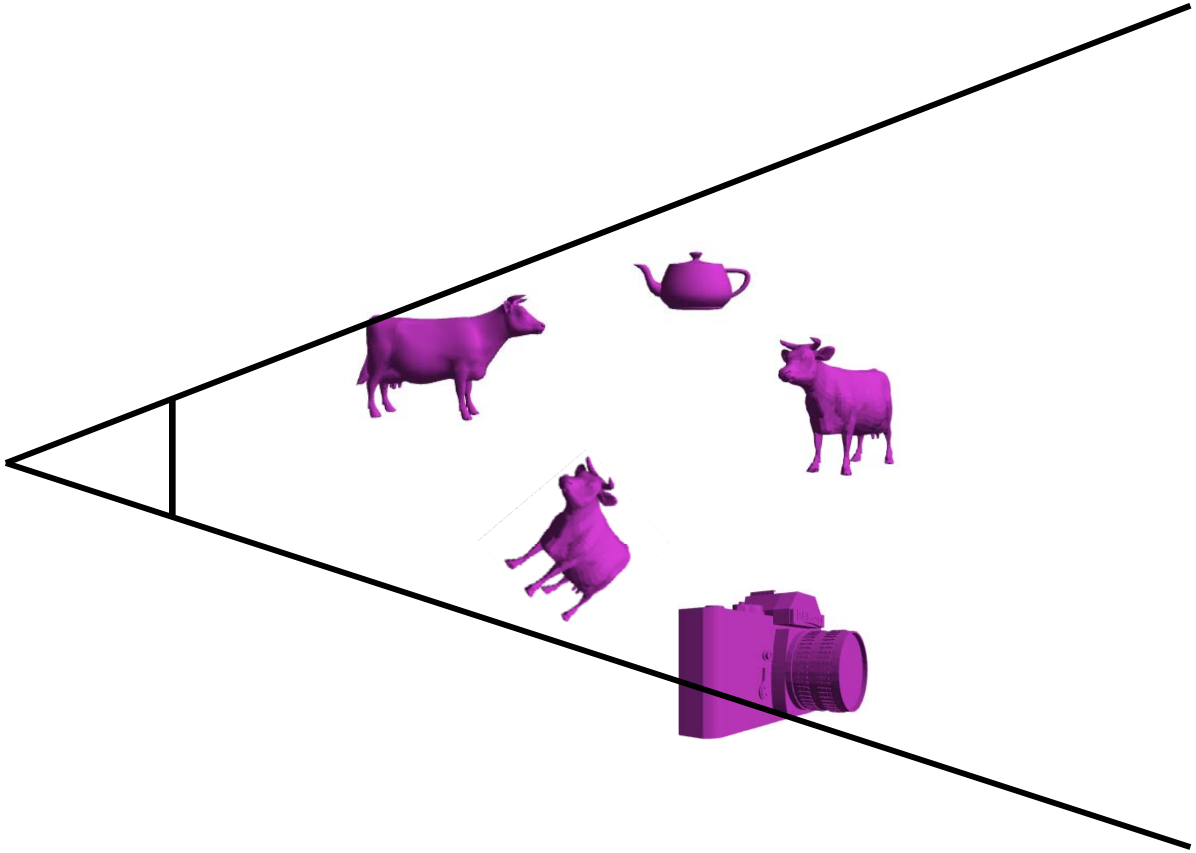
Occlusion Culling with Occlusion Queries

- **Turn off color and depth writes**
- **Render object bounding boxes with occlusion queries**
  - **An occlusion query returns the number of visible pixels**



Occlusion Culling with Occlusion Queries

- Re-enable color writes
- Render newly visible objects



Class Objectives were:

- **Understand triangle rasterization using edge-equations**
- **Understand mechanics for parameter interpolations**
- **Realize benefits of incremental algorithms**

Next Time

- **Texture mapping**