CS380: Radiometry and Rendering Equation

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Course URL:
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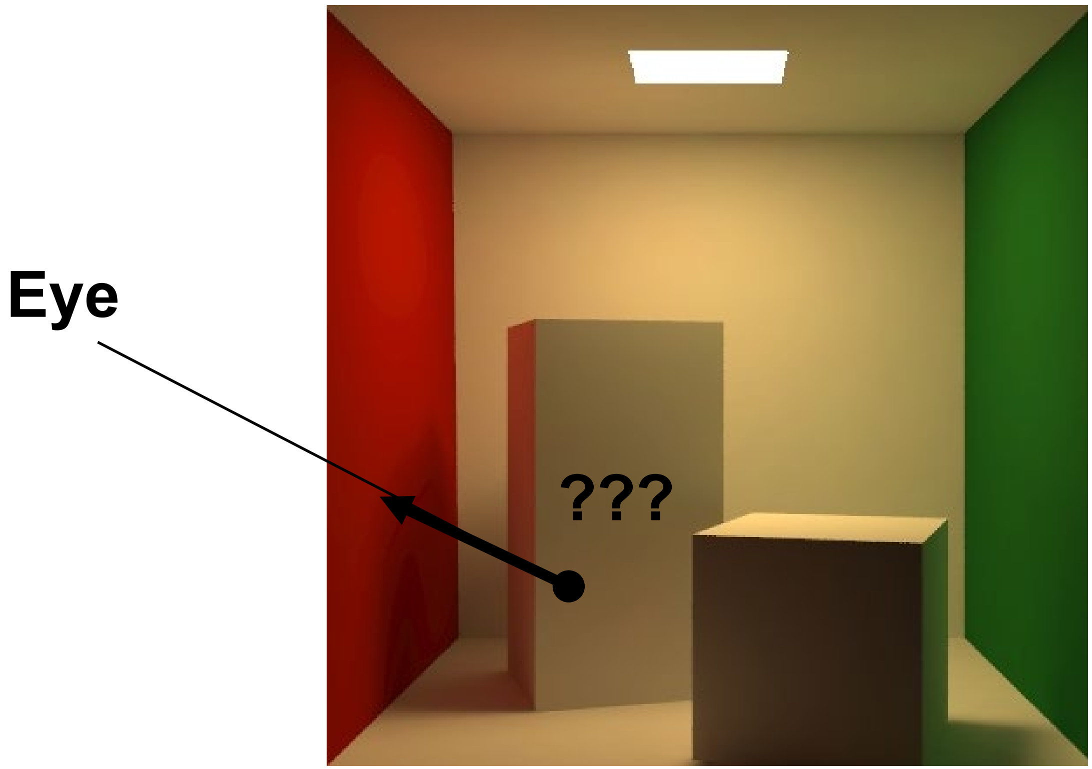
Class Objectives (Ch. 12 and 13)

- **Know terms of:**
  - **Hemispherical coordinates and integration**
  - **Various radiometric quantities (e.g., radiance)**
  - **Basic material function, BRDF**
  - **Understand the rendering equation**
- **Radiometric quantities**
  - **Briefly touched here**
  - **Refer to my book, if you want to know more**

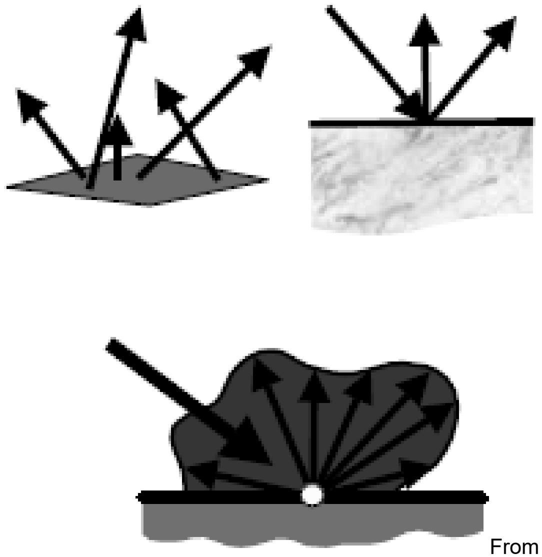
- **Last time:**
  - **Covered basic ray tracing and its acceleration data structure**

Motivation



Light and Material Interactions

- Physics of light
- Radiometry
- Material properties
- Rendering equation



From kavita's slides

Models of Light

- **Quantum optics**
  - **Fundamental model of the light**
  - **Explain the dual wave-particle nature of light**
- **Wave model**
  - **Simplified quantum optics**
  - **Explains diffraction, interference, and polarization**

- **Geometric optics**
  - **Most commonly used model in CG**
  - **Size of objects >> wavelength of light**
  - **Light is emitted, reflected, and transmitted**

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Radiometry and Photometry

- **Photometry**
  - **Quantify the perception of light energy**
- **Radiometry**
  - **Measurement of light energy: critical component for photo-realistic rendering**
  - **Light energy flows through space, and varies with time, position, and direction**
  - **Radiometric quantities: densities of energy at particular places in time, space, and direction**
  - **Briefly discussed here; refer to my book**

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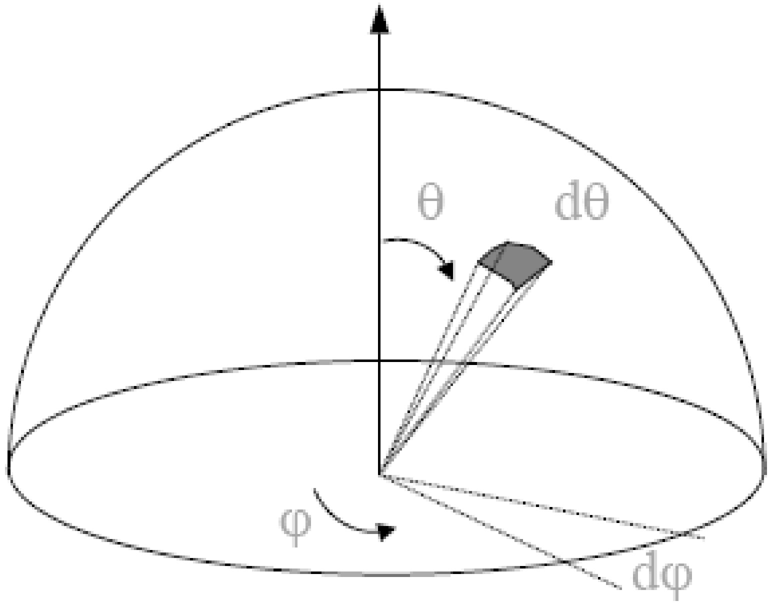
Hemispheres

- **Hemisphere**
  - **Two-dimensional surfaces**
- **Direction**
  - **Point on (unit) sphere**

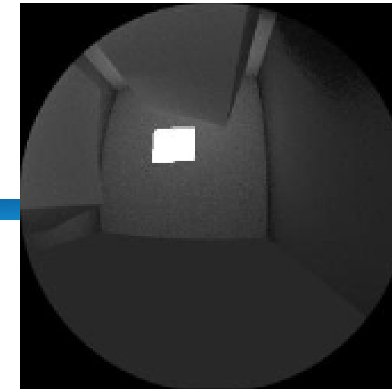


$$\theta \in [0, \frac{\pi}{2}]$$

$$\varphi \in [0, 2\pi]$$

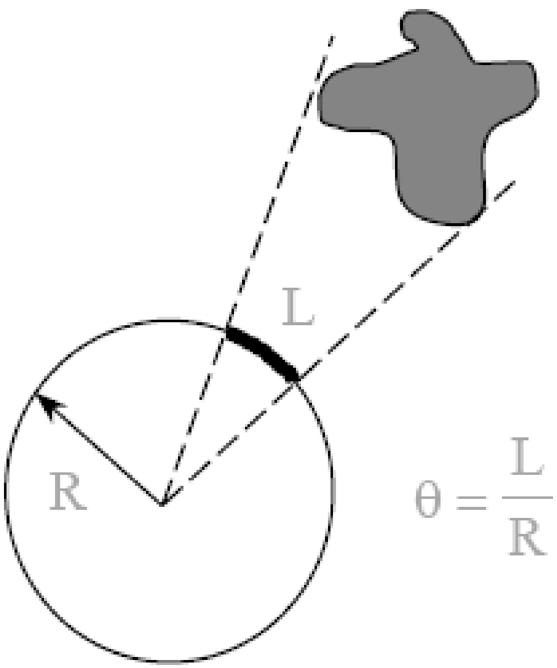
From kavita's slides

Solid Angles



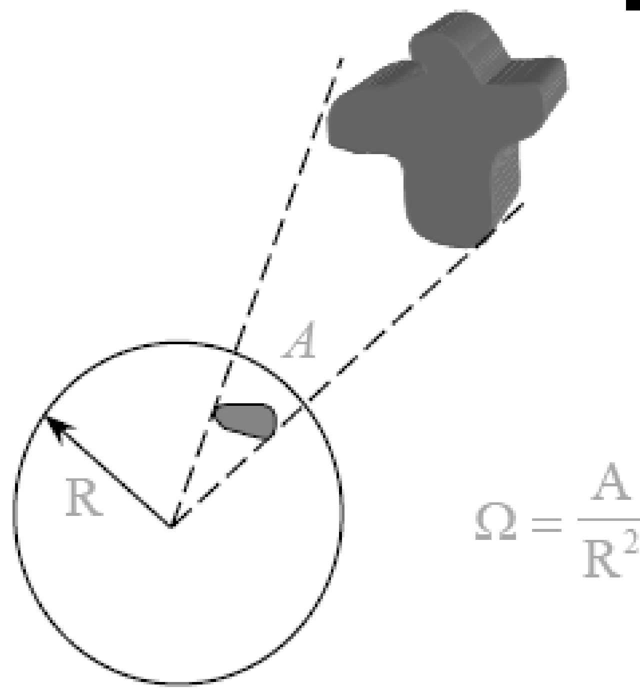
View on the hemisphere

2D



Full circle = 2pi radians

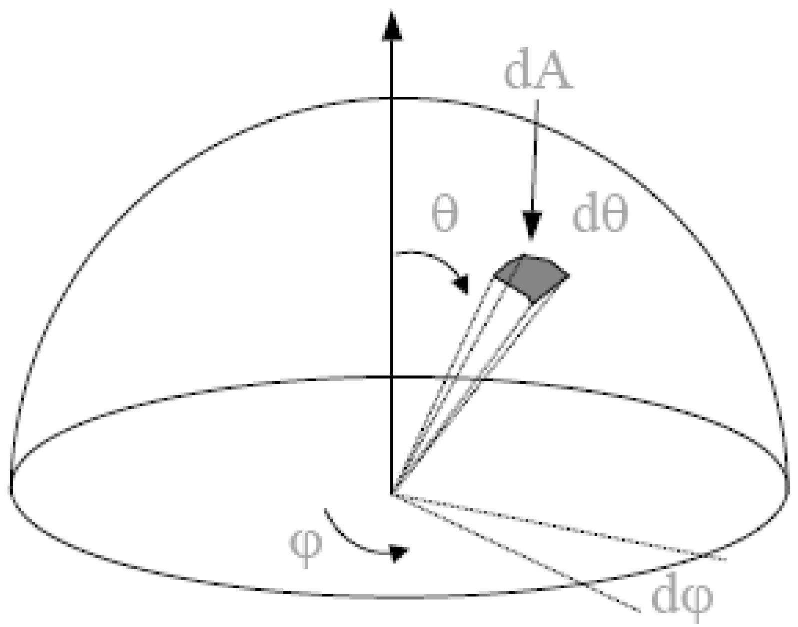
3D



Full sphere = 4pi steradians

Hemispherical Coordinates

- **Direction, Θ**
  - **Point on (unit) sphere**

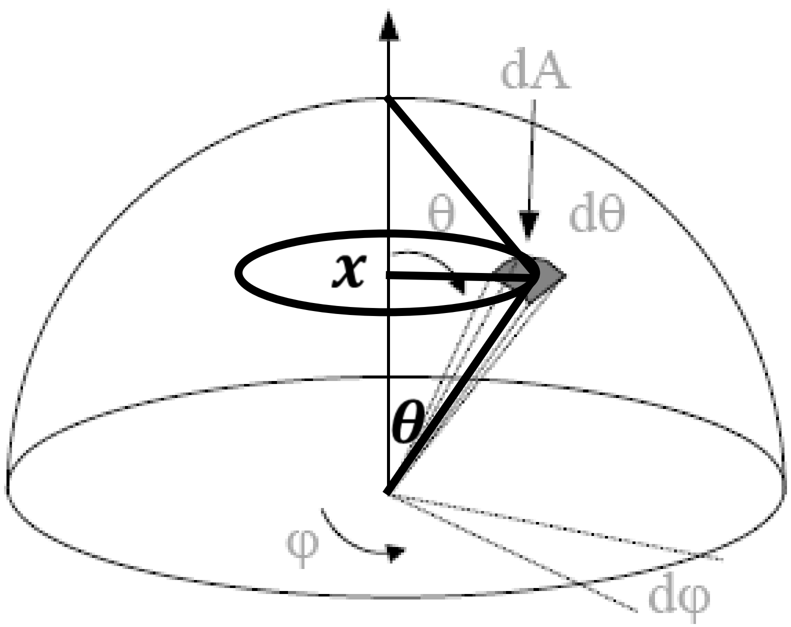


$$dA = (r \sin\theta d\varphi)(r d\theta)$$

From kavita's slides

Hemispherical Coordinates

- **Direction, Θ**
  - **Point on (unit) sphere**



$$\sin\theta = \frac{x}{r},$$

$$x = r\sin\theta$$

$$dA = (r\sin\theta\, d\varphi)(r\, d\theta)$$

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Hemispherical Coordinates

- **Differential solid angle**

$$d\omega = \frac{dA}{r^2} = \sin\theta d\theta d\varphi$$

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Hemispherical Integration

- **Area of hemispehre:**

$$\int_{\Omega_x} d\omega = \int_0^{2\pi} d\varphi \int_0^{\pi/2} \sin\theta d\theta$$

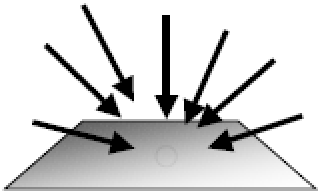
$$= \int_0^{2\pi} d\varphi \left[-\cos\theta\right]_0^{\pi/2}$$

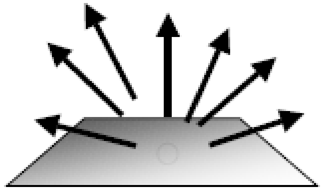
$$= \int_0^{2\pi} d\varphi$$

$$= 2\pi$$

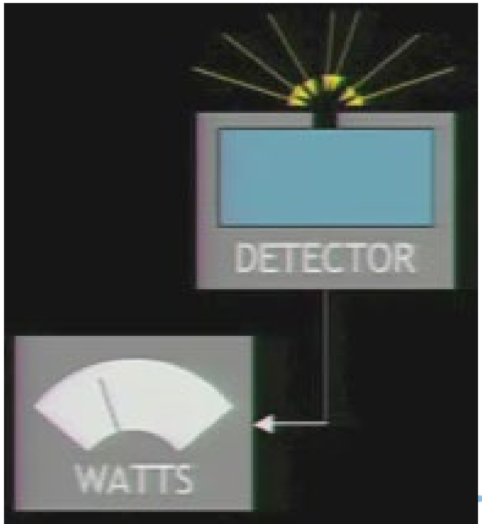
Irradiance

- Incident radiant power per unit area (dP/dA)
  - Area density of power





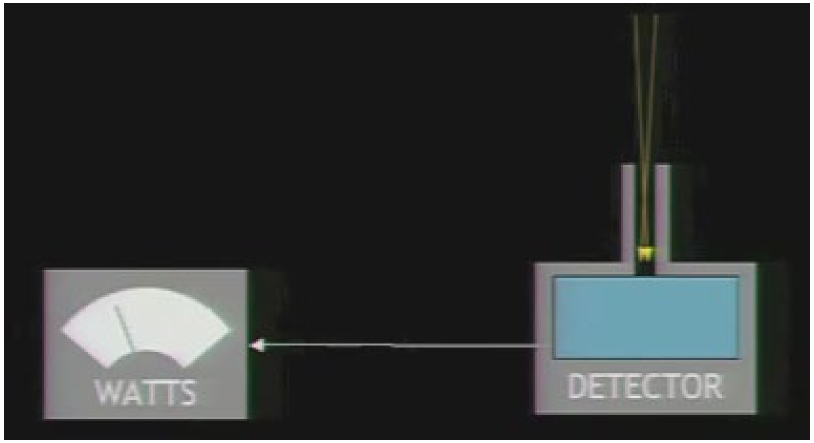
- Symbol: E, unit: W/m^2
  - Area power density exiting a surface is called radiance exitance (M) or radiosity (B)



- For example
  - A light source emitting 100 W of area 0.1 m^2
  - Its radiant exitance is 1000 W/m^2

Radiance

- **Radiant power at x in direction θ**
  - $L(x \rightarrow \Theta)$ **: 5D function**
    - **Per unit area**
    - **Per unit solid angle**

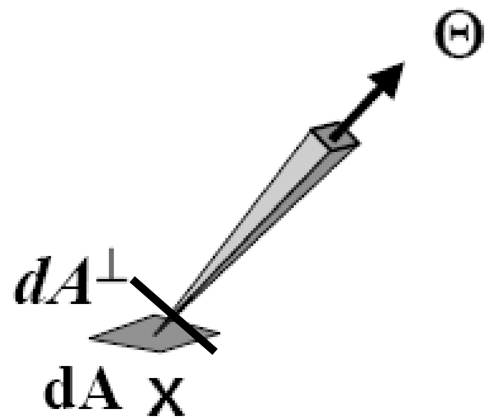


- **Important quantity for rendering**

Radiance

- **Radiant power at x in direction θ**
  - $L(x \to \Theta)$ **: 5D function**
    - **Per unit area**
    - **Per unit solid angle**

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$



  - **Units: Watt / (m^2 sr)**
  - **Irradiance per unit solid angle**
  - **2^{nd} derivative of P**
  - **Most commonly used term**

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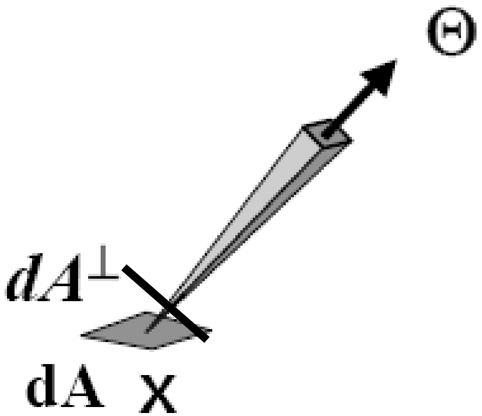
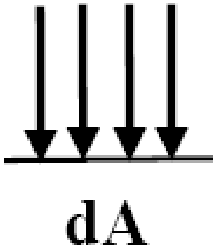
Radiance: Projected Area

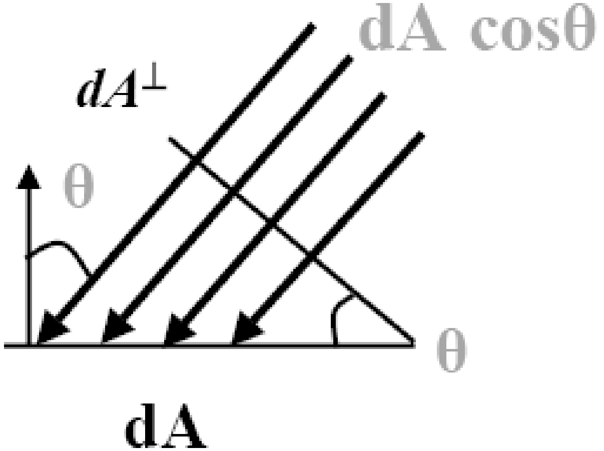
$$L(x \to \Theta) = \frac{d^2P}{dA^{\perp} d\omega_{\Theta}}$$

$$= \frac{d^2P}{d\omega_{\Theta} dA \cos\theta}$$



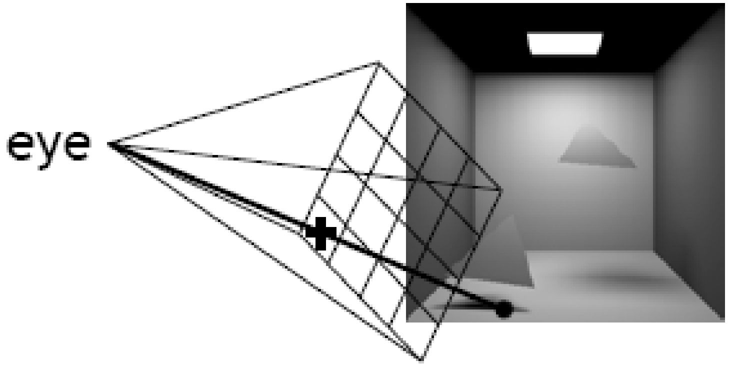
- **Why per unit projected surface area**





Sensitivity to Radiance

- **Responses of sensors (camera, human eye) is proportional to radiance**



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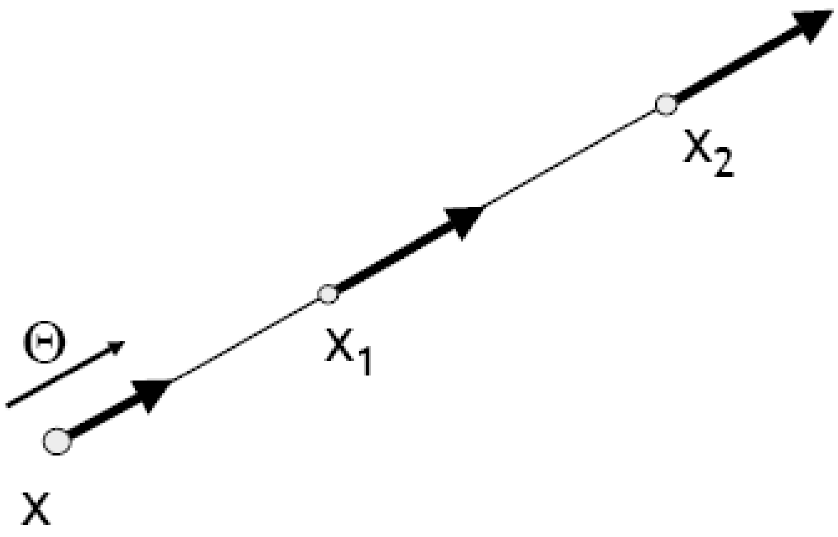
- **Pixel values in image proportional to radiance received from that direction**

Properties of Radiance

- **Invariant along a straight line (in vacuum)**



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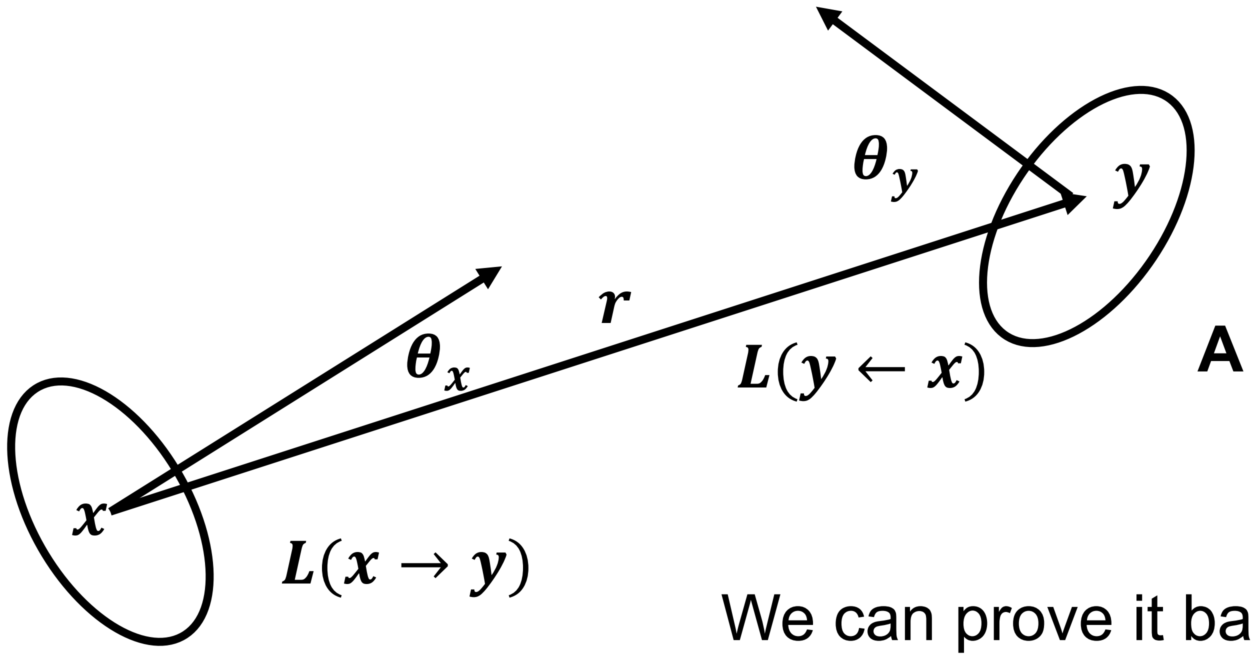
Invariance of Radiance



We can prove it based on the assumption the conservation of energy.

Relationships

- **Radiance is the fundamental quantity**

$$L(x \to \Theta) = \frac{d^2 P}{dA^{\perp} d\omega_{\Theta}}$$

- **Power:**

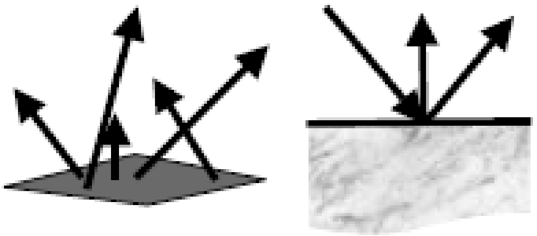
$$P = \int_{Area} \int_{\substack{Solid \\ Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta} \cdot dA$$

- **Radiosity:**

$$B = \int_{\substack{Solid \\ Angle}} L(x \to \Theta) \cdot \cos\theta \cdot d\omega_{\Theta}$$

Light and Material Interactions

- **Physics of light**
- **Radiometry**
- **Material properties**



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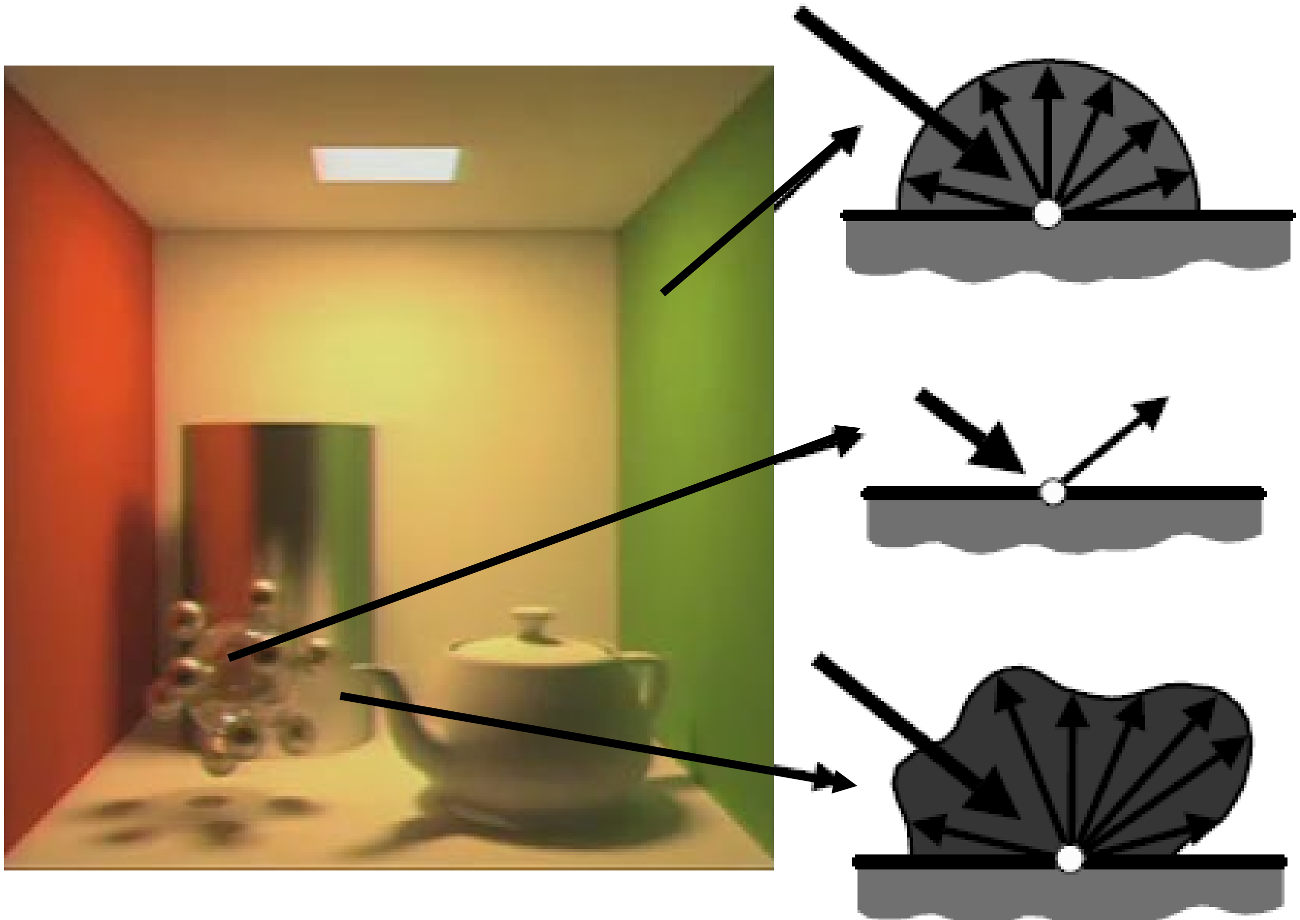
- **Rendering equation**

Materials



Ideal diffuse (Lambertian)

Ideal specular

Glossy

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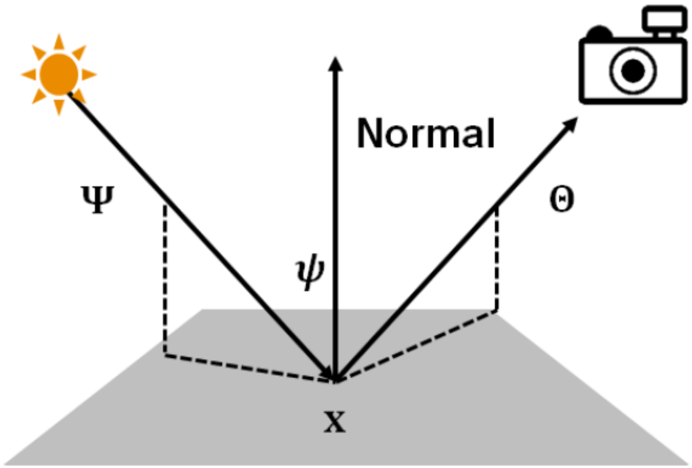
Bidirectional Reflectance Distribution Function (BRDF)

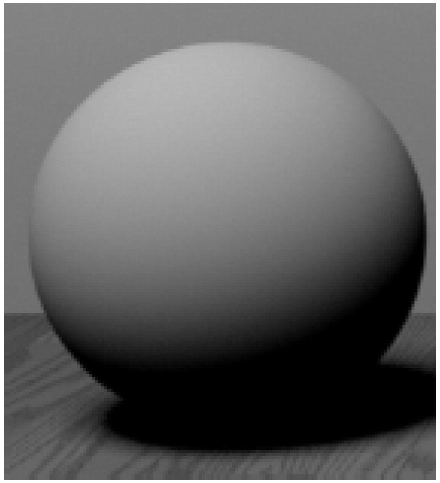


$$f_r(x, \Psi \to \Theta) = \frac{dL(x \to \Theta)}{dE(x \leftarrow \Psi)} = \frac{dL(x \to \Theta)}{L(x \leftarrow \Psi)\cos\psi\, dw_{\Psi}}$$

BRDF special case: ideal diffuse

Pure Lambertian

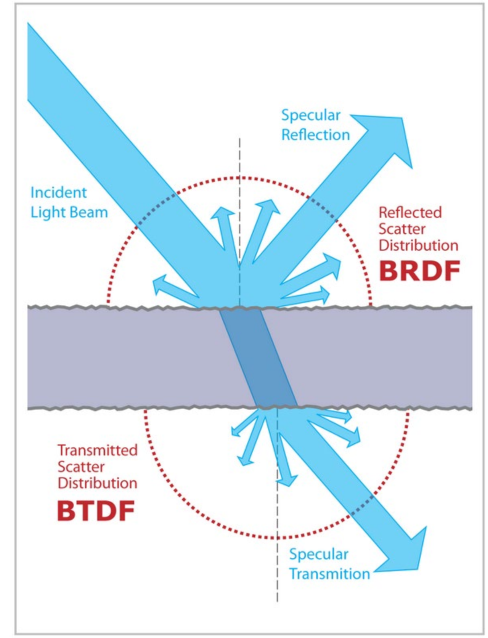
$$f_r(x, \Psi \rightarrow \Theta) = \frac{\rho_d}{\pi}$$



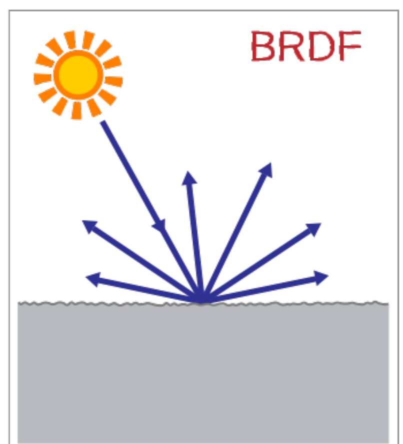
$$\rho_d = \frac{Energy_{out}}{Energy_{in}} \qquad 0 \leq \rho_d \leq 1$$

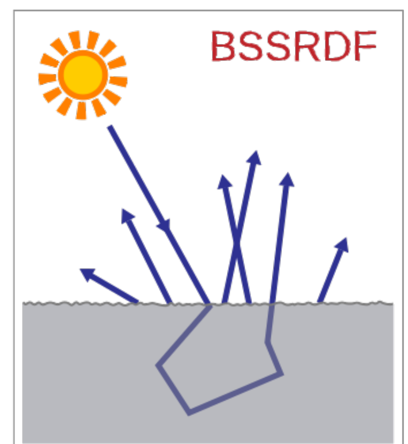
Other Distribution Functions: BxDF

- **BSDF (S: Scattering)**
  - **The general form combining BRDF + BTDF (T: Transmittance)**



- **BSSRDF (SS: Surface Scattering)**
  - **Handle subsurface scattering**





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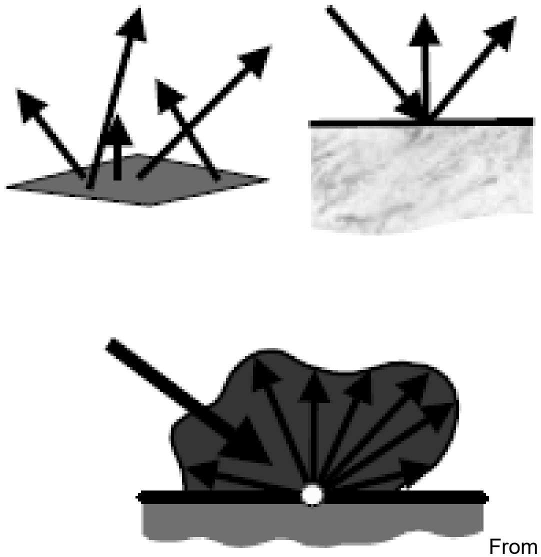
Light and Material Interactions

- **Physics of light**
- **Radiometry**
- **Material properties**



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- **Rendering equation**

Light Transport

- **Goal**
  - **Describe steady-state radiance distribution in the scene**
- **Assumptions**
  - **Geometric optics**
  - **Achieves steady state instantaneously**

Rendering Equation

- **Describes energy transport in the scene**
- **Input**
  - **Light sources**
  - **Surface geometry**
  - **Reflectance characteristics of surfaces**
- **Output**
  - **Value of radiances at all surface points in all directions**

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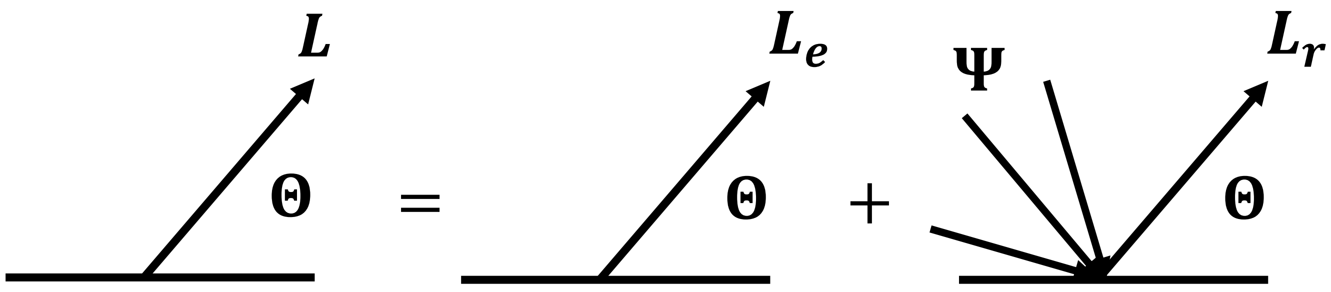
Rendering Equation



$$L(x \to \Theta) = L_e(x \to \Theta) \quad + \quad L_r(x \to \Theta)$$

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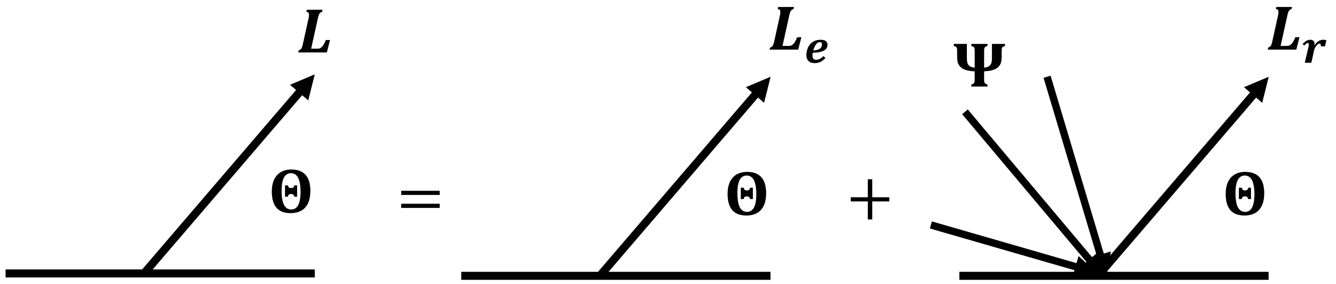
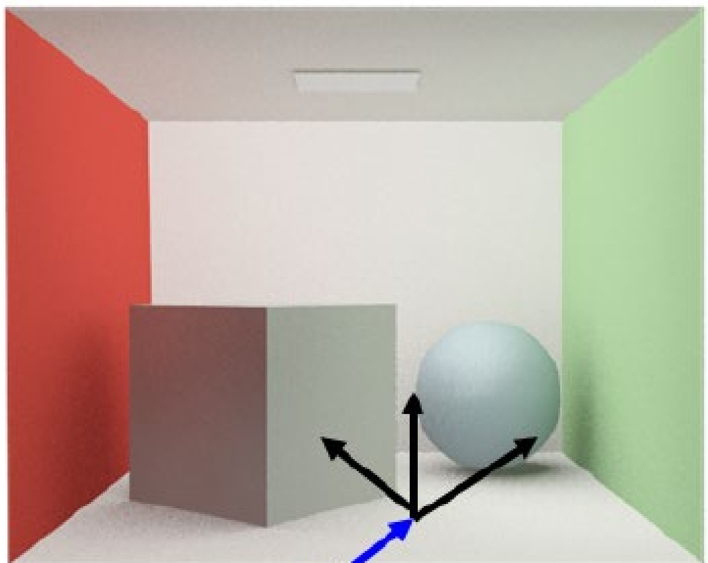
Rendering Equation

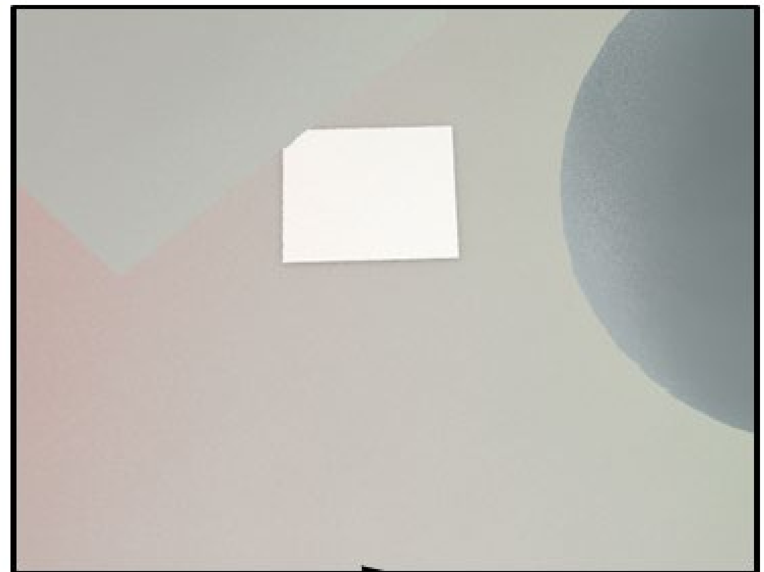


$$L_r(x \rightarrow \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \rightarrow \Theta) \cos\theta_x dw_{\Psi},$$

- **Applicable to all wave lengths**

Rendering Equation



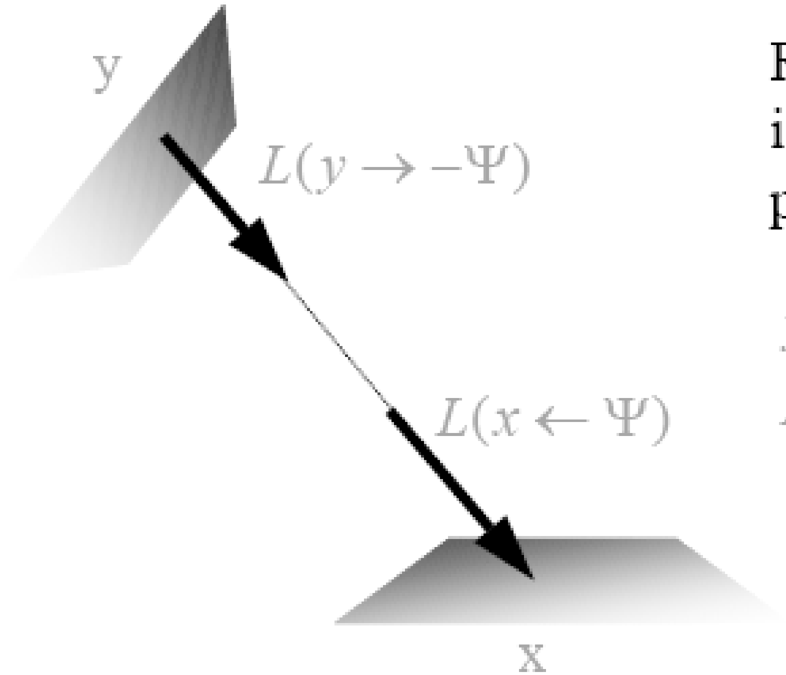


Incoming radiance on the hemisphere

$$L_r(x \to \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \to \Theta) \cos\theta_x dw_{\Psi}$$

Rendering Equation: Area Formulation

$$L(x \to \Theta) = L_e(x \to \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



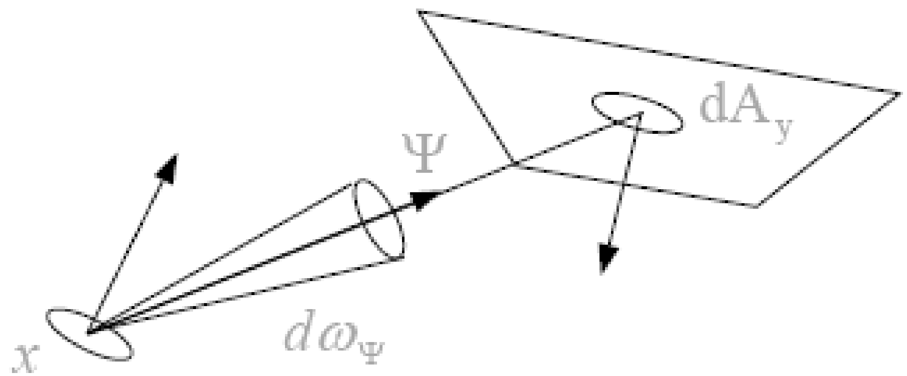
Ray-casting function: what is the nearest visible surface point seen from x in direction Ψ?

$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \to -\Psi)$$

Rendering Equation

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$



$$y = vp(x, \Psi)$$

$$L(x \leftarrow \Psi) = L(vp(x, \Psi) \rightarrow -\Psi)$$

$$d\omega_\Psi = \frac{dA_y \cos\theta_y}{r_{xy}^2}$$

Rendering Equation: Visible Surfaces

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\Omega_x} f_r(\Psi \leftrightarrow \Theta) \cdot L(x \leftarrow \Psi) \cdot \cos\theta_x \cdot d\omega_\Psi$$

Coordinate transform

$$L(x \rightarrow \Theta) = L_e(x \rightarrow \Theta) + \int_{\substack{y \text{ on} \\ \text{all surfaces}}} f_r(\Psi \leftrightarrow \Theta) \cdot L(y \rightarrow -\Psi) \cos\theta_x \cdot \frac{\cos\theta_y}{r_{xy}^2} \cdot dA_y$$

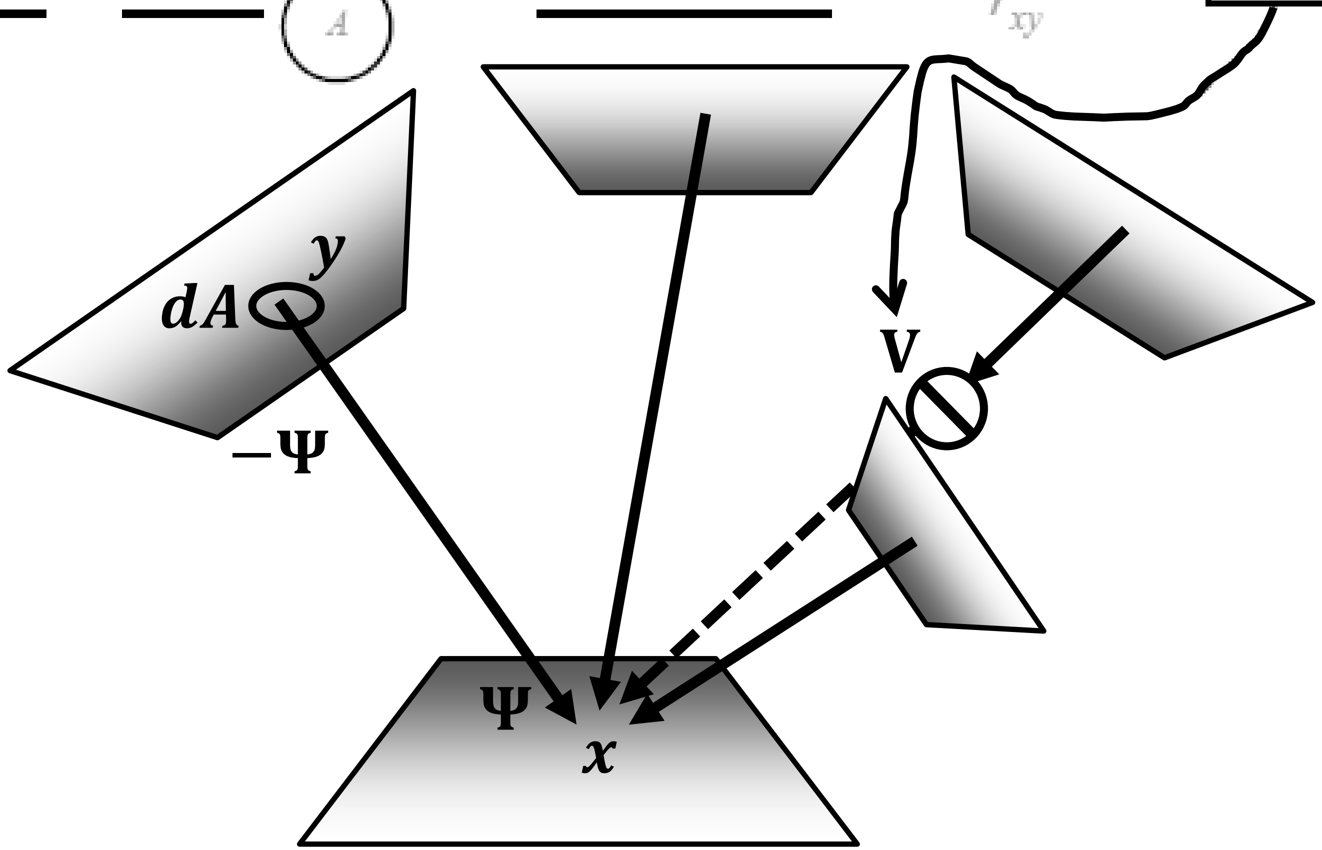
$$y = vp(x, \Psi)$$

Integration domain = visible surface points y

- Integration domain extended to ALL surface points by including visibility function

Rendering Equation: All Surfaces

$$L(x \rightarrow \Theta) = L_e(\ldots) + \int_A f_r(\ldots) \cdot L(y \rightarrow -\Psi) \cdot \frac{\cos\theta_x \cdot \cos\theta_y}{r_{xy}^2} \cdot V(x,y)\, dA_y$$



Two Forms of the Rendering Equation

- **Hemisphere integration**

$$L_r(x \to \Theta) = \int_{\Psi} L(x \leftarrow \Psi) f_r(x, \Psi \to \Theta) \cos\theta_x dw_{\Psi}$$

- **Area integration (used as the form factor)**

$$L_r(x \to \Theta) = \int_{A} L(y \to -\Psi) f_r(x, \Psi \to \Theta) \frac{\cos\theta_x \cos\theta_y}{r_{xy}^2} V(x, y) dA,$$

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Class Objectives (Ch. 12 & 13) were:

- Know terms of:
  - Hemispherical coordinates and integration
  - Various radiometric quantities (e.g., radiance)
  - Basic material function, BRDF
  - Understand the rendering equation

Next Time

- Monte Carlo rendering methods